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Parametric instability of a rotating truncated conical shell subjected to periodic axial loads

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ABSTRACT

Parametric instability of a rotating truncated conical shell subjected to periodic axial loads is studied in the paper. Through deriving accurate expressions of inertial force and initial hoop tension, a rotating conical shell model is presented based upon the Love's thin shell theory. Considering the periodic axial loads, equations of motion of the system with periodic stiffness coefficients are obtained utilizing the generalized differential quadrature (GDQ) method. Hill's method is introduced for parametric instability analysis. Primary instability regions for various natural modes are computed. Effects of rotational speed, constant axial load, cone angle and other geometrical parameters on the location and width of various instability regions are examined.

1. Introduction

Shell structures have many applications in civil, mechanical and aerospace engineering. Such elements subjected to in-plane periodic forces may undergo unstable transverse vibrations, leading to parametric instability, due to certain combinations of the values of load parameters and natural frequency of transverse vibration. The parametric instability of shell structures has been extensively studied over the last four decades. Early research progress could be found in Bolotin (1964), Ewan-Iwanowski (1965), Ibrahim (1978) and Simitses (1987). Recently, in depth reviews on dynamic stability behavior of plates and shells with various geometries, boundary conditions and load types were published by Sahu and Datta (2007).

As the literature shown, many studies were focused on cylindrical shells, while the parametric instability of conical shells received relatively less attention. Kornecki (1966) first studied the dynamic stability of truncated conical shells under pulsating pressure. The influence of deformations prior to instability on the dynamic stability of conical shells was studied by Tani (1976a,b) for the system under periodic axial loading and periodic pressure, respectively. Later, Tani (1981) also investigated the dynamic stability characteristics of conical shells under pulsating torsion excitations. Massalas et al. (1981) employed Galerkin's method to reduce the base equations but used Bolotin's method to obtain the principal instability regions. In their investigation, the formulation considered the clamped conical shells with variable modulus of elasticity. Using the Marguerre type dynamic equations, Ye (1997) analyzed the non-linear vibration and dynamic instability of thin shallow conical shells subjected to periodic transverse and in-plane loads. Ng et al. (1999) utilized the generalized differential quadrature (GDQ) method to examine the effects of boundary conditions on the parametric instability of truncated conical shells under periodic edge loading. Studies have also been carried out on orthotropic truncated conical shells. Ganapathia et al. (1999) conducted dynamic stability analysis of laminated conical shells under periodic in-plane load. The influences of various parameters such as cone orthotropicity, cone angle, ply-angle and elastic edge restraint on dynamic stability were brought out. Utilizing the perturbation method, thermally induced dynamic instability of laminated composite conical shells was investigated by Wu and Chiu (2002). Recently, Sofiyev (2004, 2007, 2009) carried out a series of work on the nonlinear stability and buckling behaviors of functionally graded truncated conical shells.

The articles mentioned above concentrated mainly on the non-rotating conical shell. To the knowledge of the authors, no publication is available in the open literature that reports the effect of rotation on the dynamic stability of rotating conical shells. For the rotating cylindrical shell under periodic axial loads, Ng et al. (1998) and Liew et al. (2006) have reported that the coriolis and centrifugal forces

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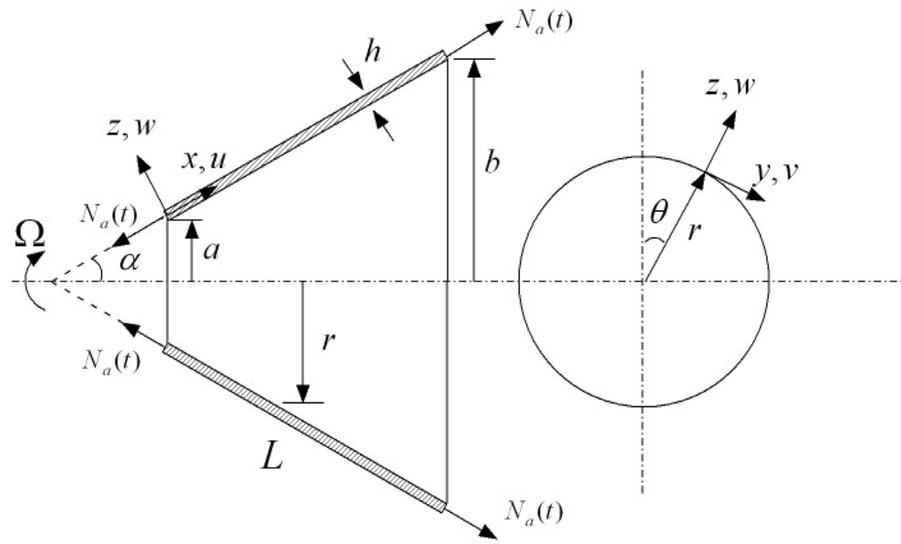

Fig. 1. Geometry and co-ordinate of a rotating thin truncated circular conical shell under periodic loading in the meridional direction.

induced by rotation have significant impacts on the instability regions. Lam and Hua (1997) pointed out that the amount of analysis required to extend the study from rotating cylindrical shell to rotating conical shell is quite considerable. Also, the conical shell is a more general type of shell structures. Thus, it is necessary and meaningful to investigate the effect of rotation on the parametric instability of rotating conical shells under periodic axial loads.

The GDQ method was first presented by Shu and Richards (1992) to directly solve the governing equations of engineering problems. Presently, the GDQ method has been widely used for vibration analysis of shell structures (Loy et al., 1997; Hua and Lam, 2000; Lam and Hua, 2000). In this paper, the method will be utilized for establishing the vibration model of rotating conical shell. As long as the rotation is considered, the rotating conical shell under periodic axial loads becomes a parametrically excited gyroscopic system. The Bolotin's method (Bolotin, 1964) could not be used for instability analysis for such system, as the assumption of Floquet multipliers in Bolotin's method cannot be satisfied for the gyroscopic system (Pei, 2009; Pei and Tan, 2009). Thus, the Hill's method will be used for parametric instability analysis.

The content of this paper is organized as follows: first, the equations of motion of a rotating conical shell subjected to periodic axial loads are derived based upon the Love's thin shell theory and GDQ method. Then, the Hill's method is introduced for parametric instability analysis. Based upon these, the instability regions for various natural modes are computed and discussed. Effects of rotational speed, constant axial load, cone angle and other geometrical parameters on the location and width of various instability regions are also examined in detail. Finally, some conclusions are given.

2. Formulations for the rotating conical shell subjected to periodic axial load

The truncated conical shell is isotropic and has Young's modulus E, mass density ρ and Poisson's ratio ν. Fig. 1 shows the geometry and co-ordinate system for a truncated circular conical shell rotating about its symmetrical and horizonal axis at an angular velocity Ω. In the figure, α is the cone angle, L the length, h the thickness, and a and b are the radii at the two ends. The reference surface of the conical shell is taken to be at its middle surface where an orthogonal co-ordinate system (x, θ, z) is fixed, and $r = r(x)$ is a radius at any co-ordinate point (x, θ, z). The deformations of the rotating conical shell in the meridional x, circumferential θ and normal z directions are defined by u, v, w, respectively.

The rotating conical shell is subjected to periodic loading in the meridional direction as shown in Fig. 1. The time-dependent axial load $N_a(t)$ is assumed to be a small and sinusoidal perturbation superimposed upon a constant load as

$$N_a(t) = \eta_a N_{cr}(1 + \epsilon \cos \omega_a t) \tag{1}$$

where η_a and ϵ, respectively, denote the relative amplitudes of constant and perturbed loads. N_{cr} represents the buckling load of intermediate length cylindrical shells given by Timoshenko and Gere (1961), i.e. $N_{cr} = Eh^2/(R\sqrt{3(1-\nu^2)})$ in which R is the radius of the cylindrical shell. In present analysis, we set $R = a$. The governing equations of motion of the rotating conical shell have been given in Lam and Hua (1997). However, the expressions of inertial force and initial hoop tension are found inadequate and some terms are missing. In the following, the actual expressions of both inertial and initial hoop tension are derived, and corrected equations of motion for the rotating conical shell are then presented.

2.1. Inertial forces, initial hoop tension and translational force induced by axial loads

A unit vector $(\vec{i}, \vec{j}, \vec{k})$ is defined for the co-ordinate (x, θ, z). The position vector of any point on the shell from the rotating axis is expressed as

$$\mathbf{r} = u\vec{i} + v\vec{j} + w\vec{k} \tag{2}$$