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Energy harvesting of monostable Duffing oscillator under Gaussian white noise excitation

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ABSTRACT

Energy harvesting of monostable Duffing oscillator with piezoelectric coupling under Gaussian white noise excitation is investigated. Based on the Fokker–Plank–Kolmogorov equation of piezoelectric coupling systems, the statistical moments of the response are derived from the Van Kampen expansion. The effects of the spectral density of the random excitation and the coefficient of cubic nonlinearity on the expected response moments are analyzed. Some numerical examples are presented to demonstrate the effects of excitation spectral density, coefficient of cubic nonlinearity and initial conditions on the output voltage.

1. Introduction

Energy harvesting from ambient waste energy for the purpose of running low-powered electronics has emerged as a prominent research area and continues to grow at rapid pace. The area of vibration-based energy harvesting encompasses mechanics, materials science, and electrical circuitry. One of the most studied areas is the use of the piezoelectric effect to convert ambient vibration into useful electrical energy.

Piezoelectric energy harvesting can be configured in many different ways that prove useful in power harvesting applications. The rapid growth of research being performed in the field of piezoelectric energy harvesting has resulted in significant improvements to various energy scavenging techniques. There are several excellent and comprehensive survey papers, notably Sodano et al. (2004), Anton and Sodano (2007), Priya (2007), Zhu et al. (2010), reviewing the state of the art in different time phases of investigations related to piezoelectric energy harvesting. Recently, there are two monographs have been published to present the current state of knowledge in energy harvesting technologies (Priya and Inman, 2009) and piezoelectric energy harvesting (Erturk and Inman, 2011).

Concentrating on discrete and continuous systems at the resonance under a harmonic excitation, most works on energy harvesting take the deterministic approaches. However, uncertainty is inherent in most real-world systems, and the uncertainty may seriously change the behavior of vibration. There are some researches via stochastic approaches. Establishing the closed-form expressions for output power, proof mass, displacement, and optimal load for linear energy harvesters driven by broadband random vibrations, Halvorsen (2008) demonstrated that the output power has a different optimum for broadband excitations from that for sinusoidal excitations. McInnes et al. (2008) employed the stochastic resonance to enhance vibration energy harvesting and revealed numerically the significant enhancement without any periodic forcing. Introducing the nonlinearity resulted from magnetic interactions Cottone et al. (2009) found numerically and experimentally the nonlinearity improve the vibration-based energy harvesters. Based on a linear model of the piezoelectric material along the d-33 direction, Adhikari et al. (2009) determined the mean power acquired from a piezoelectric vibration-based energy harvesting circuit subjected to stationary Gaussian white noise. Calculating the response of uni-modal electromagnetic Duffing-type harvesters to Gaussian white and colored excitations Daqaq (2010) found that, under white excitations, nonlinearities in the damping and the inertia but not the stiffness enhance the expected value of the output power, and, under colored excitations, the nonlinearity decreases the expected value of the output power regardless of the bandwidth or the center frequency of the excitation. Proposing transduction of

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a bistable inductive generator driven by white and exponentially correlated Gaussian noise, Daqaq (2011) derived an approximate expression for the mean power under exponentially correlated noise and demonstrated the existence of an optimal potential shape maximizing the output power.

It should be remarked that there are different types of electrical circuit equations used in piezoelectric energy harvesting. duToit et al. (2005) first proposed a coupled electromechanical equation for lumped-parameter piezoelectric energy harvesters. As the motion equation of the harvester and its electromechanical equation cannot be directly decoupled, the system is with 1.5 degrees-of-freedom. This type of the electrical circuit equation has been widely used (Erturk and Inman, 2011; Halvorsen, 2008; McInnes et al., 2008; Cottone et al., 2009). Daqaq (2010, 2011) and Green et al. (2012) introduced an uncoupled electrical circuit equation, the electromechanical equation can be converted into first-order differential equation. Therefore, the system is with single-degree-of-freedom. Triplett and Quinn (2009) considered a nonlinear piezoelectric coupling relationship on the performance of a vibration-based energy harvester. Adhikari et al. (2009) reported an electrical circuit equation with an inductor, where the electrical equation is second-order differential equation. Thus, the system is with 2 degrees-of-freedom. This paper investigated the lumped-parameter model of a piezoelectric energy harvester which is essentially a 1.5 degree-of-freedom system. So far, to the authors' best knowledge, there are no investigations on the FPK equations of monostable Duffing oscillator with 1.5 degree-of-freedom system. To address the lacks of research in the aspect, the present work treats the monostable Duffing oscillator with piezoelectric coupling under Gaussian white noise excitation.

The paper is organized as follows. In Section 2, the Fokker–Plank–Kolmogorov equation of piezoelectric coupling systems is derived from the equations of piezoelectric coupling under Gaussian white noise excitation. The Van Kampen expansion is applied to determine the statistical moments of the response. The effects of the spectral density of the random excitation and the coefficient of cubic nonlinearity on the expected response moments are analyzed. In Section 3, numerical simulations are presented to demonstrate the effects of the bandwidth of excitation amplitude, the coefficient of cubic nonlinearity and initial conditions on output voltage. Section 4 ends the paper with concluding remarks.

2. The response moments to Gaussian white excitation

The lumped-parameter equations of a piezoelectric energy harvester with cubic nonlinearity in the displacement term under harmonic excitation have been given by Erturk and Inman (2011). However, here we consider the case of stochastic excitation rather than harmonic excitation. The lumped-parameter equations of a piezoelectric energy harvester with cubic nonlinearity under Gaussian white noise excitation can be given as

$$\ddot{x} + 2\varepsilon\mu\omega_0\dot{x} + \omega_0^2 x + \varepsilon\alpha x^3 - \varepsilon\chi\upsilon = \varepsilon\xi(t) \quad (1)$$

$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0 \quad (2)$$

where x is the displacement response, υ is the dimensionless voltage response across the external electrical load, ω_0 is the undamped fundamental natural frequency, χ is the dimensionless piezoelectric coupling term in the mechanical equation, λ is the reciprocal of the dimensionless time constant of the resistive–capacitive circuit, and κ is the dimensionless piezoelectric coupling term in the electrical equation. Furthermore, ε is a small bookkeeping parameter and μ is a mechanical damping term, $\xi(t)$ is Gaussian white noise process with zero mean and autocorrelation function.

$$\left\langle \xi(t)\xi(t+\tau) \right\rangle = 2\pi S_0 \delta(\tau) \quad (3)$$

where $\langle\ \rangle$ denotes the expected value, S_0 is the spectral density of the excitation, and δ is the dirac-delta function.

Eqs. (1) and (2) can be converted into Itô differential equations

$$\begin{aligned} dx_1 &= x_2 dt \\ dx_2 &= -(2\varepsilon\mu\omega_0 x_2 + \omega_0^2 x_1 + \varepsilon\alpha x_1^3 - \varepsilon\chi x_3)dt + \varepsilon\sqrt{2S}dB(t) \\ dx_3 &= -(\lambda x_3 + \kappa x_2)dt \end{aligned} \quad (4)$$

where $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \upsilon$, $S = \pi S_0$, $B(t)$ is a Brownian motion process. The joint PDF, $P(x_1, x_2, x_3, t)$, of the response can be obtained by solving the FPK equation which can be expressed for system as

$$\begin{aligned} \frac{\partial P}{\partial t} &= -\frac{\partial}{\partial x_1}(x_2 P) + \frac{\partial}{\partial x_2}[(2\varepsilon\mu\omega_0 x_2 + \omega_0^2 x_1 + \varepsilon\alpha x_1^3 - \varepsilon\chi x_3)P] \\ &\quad + \frac{\partial}{\partial x_3}[(\lambda x_3 + \kappa x_2)P] + \varepsilon^2 S \frac{\partial^2 P}{\partial x_2^2} \end{aligned} \quad (5)$$

subjected to the boundary conditions $P(-\infty, t) = P(+\infty, t) = 0$. Even in the steady-state case, an exact solution of Eq. (5) is not attainable. For small nonlinearities and mean square values of the x_i's, one can use the Van Kampen expansion (Rodriguez and Kampen, 1976) to obtain an approximate solution of Eq. (5). In the Van Kampen expansion, which was introduced in the context of some statistical physics problem, the variables are expanded in a successive powers of the excitation's spectral density S. That is,

$$\begin{aligned} x_1 &= S^{1/2}\eta_1 + O(S^{3/2}) \\ x_2 &= S^{1/2}\eta_2 + O(S^{3/2}) \\ x_3 &= S^{1/2}\eta_3 + O(S^{3/2}) \end{aligned} \quad (6)$$

The reason for expanding x_i in orders of $S^{1/2}$ stems from our previous knowledge that mean square value of x_i or $\left\langle x_i^2 \right\rangle$, which is a measure of the response amplitude, will turn out to be proportional to S. With this expansion, the PDF becomes a function of the new variables η_i as

$$P(x_1, x_2, x_3, t) = P(S^{1/2}\eta_1, S^{1/2}\eta_2, S^{1/2}\eta_3, t) = G(\eta_1, \eta_2, \eta_3, t) \quad (7)$$

In terms of the new PDF, the FPK equation becomes

$$\begin{aligned} \frac{\partial G}{\partial t} &= -\eta_2\frac{\partial G}{\partial \eta_1} + 2\varepsilon\mu\omega_0\frac{\partial(\eta_2 G)}{\partial \eta_2} + (\omega_0^2\eta_1 + \varepsilon\alpha S\eta_1^3 - \varepsilon\chi\eta_3)\frac{\partial G}{\partial \eta_2} \\ &\quad + \lambda\frac{\partial(\eta_3 G)}{\partial \eta_3} + \kappa\eta_2\frac{\partial G}{\partial \eta_3} + \varepsilon^2\frac{\partial^2 G}{\partial \eta_2^2} + O(S^{3/2}) \end{aligned} \quad (8)$$

subjected to the boundary conditions $G(-\infty, t) = G(+\infty, t) = 0$. The equations governing the response statistics (statistical moments) will be established. For a general function $\phi(\eta_1, \eta_2, \eta_3)$, the response statistics $\left\langle \phi \right\rangle$ can be obtained by multiplying both sides of Eq. (8) by ϕ and integrating by parts over the entire space; that is

$$\begin{aligned} &\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\phi\frac{\partial G}{\partial t}d\eta_1 d\eta_2 d\eta_3 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \\ &\times\phi\left[-\eta_2\frac{\partial G}{\partial \eta_1} + 2\varepsilon\mu\omega_0\frac{\partial(\eta_2 G)}{\partial \eta_2} + (\omega_0^2\eta_1 + \varepsilon\alpha S\eta_1^3 - \varepsilon\chi\eta_3)\frac{\partial G}{\partial \eta_2}\right. \\ &\left. +\lambda\frac{\partial(\eta_3 G)}{\partial \eta_3} + \kappa\eta_2\frac{\partial G}{\partial \eta_3} + \varepsilon^2\frac{\partial^2 G}{\partial \eta_2^2}\right]d\eta_1 d\eta_2 d\eta_3. \end{aligned} \quad (9)$$