Mechanics Research Communications 44 (2012) 15–23

Contents lists available at SciVerse ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Transient probability density of nonlinear multi-degree-of-freedom system with time delay

T. Jin, X.L. Jin*, Y. Wang, Z.L. Huang

Department of Engineering Mechanics, Zhejiang University, Hangzhou 310027, PR China

ARTICLE INFO

Article history:
Received 27 November 2011
Received in revised form 31 March 2012
Available online 15 May 2012

Keywords:
Transient probability density
Time delay
Nonlinear multi-degree-of-freedom system
Stochastic averaging
Galerkin method

ABSTRACT

The transient probability densities of nonlinear multi-degree-of-freedom systems with time delay are investigated. The system is firstly approximated by the corresponding non-time-delay system through appropriate relations between the current states and the delay states. Stochastic averaging is adopted to reduce the dimension of the equivalent system, and the corresponding Fokker–Planck–Kolmogorov equation with regard to transient probability density is obtained. This equation is solved by expressing the transient probability density as multiple series in terms of a set of properly state-dependent orthogonal basis functions with time-dependent coefficients. Two examples are given to illustrate the accuracy and efficacy of the proposed procedure.

1. Introduction

Realistic structures subject to random loads are generally modeled as stochastically excited nonlinear multi-degree-of-freedom (MDOF) systems. In recent decades, the responses of nonlinear stochastic MDOF systems have been extensively studied and several procedures, such as the Fokker–Planck–Kolmogorov (FPK) equation, moment closure, perturbation, equivalent linearization method, equivalent nonlinear system method and stochastic averaging have been developed to evaluate the system responses (Lin and Cai, 2004; Zhu, 2006).

The probability density of stochastic responses is the whole probabilistic description and all the information of stochastic responses can be expressed through it. Thus, the evaluation of probability density is the ultimate target to determine the system responses. The existing methods to predict the approximate stationary probability density functions have been greatly developed (Cai et al., 1992; Hasofer and Grigoriu, 1995; Bernard and Wu, 1998; Bellizzi and Bouc, 1999; To, 2005; Zhu, 2006; Crandall, 2006; Floris, 2010) and the exact stationary probability densities of several classes of nonlinear systems subject to Gaussian white noise excitations have been also studied (Soize, 1994; Wang and Zhang, 2000; Zhu, 2006). From these obtained stationary probability densities, only the information of stationary responses can be evaluated. The determination of transient probability density, however, is quite challenging due to its dependence on time. Most of the existing works are dedicated to the direct evaluation of the first and second moments, certainly higher order moments can be obtained by some methods such as the response moment approach. The progresses within this theme are so far predominantly based on the equivalent linearization method and the Monte Carlo simulation. Ohtori and Spencer (2002) obtained a recursive expression for the mean and covariance responses of MDOF systems by using the linear transformation techniques and a semi-implicit integration algorithm. Saha and Roy (2007) proposed the Girsanov linearization method for stochastically driven nonlinear oscillators. To our best knowledge, the results on the evaluation of transient probability densities of nonlinear MDOF systems are quite few (Jin and Huang, 2010).

The control of MDOF systems is an active research area in structural and control engineering. The time delay of the control forces, however, is unavoidable due to the time consumption of sensors, processor and actuators. Time delay will deteriorate the control performance and may induce the system instability. Many researchers have studied the responses of deterministic systems with time delay (Hu and Wang, 2002). The evolutions of mean values and mean square values of the dynamic responses of time-delayed systems under stochastic excitation have been also studied (Di Paola and Pirrotta, 2001; Elbeyli et al., 2005). Guillouzic et al. (1999) obtained the steady-state probability density of stochastic delay differential equations by using a small-delay approximation. Liu and Zhu (2007) obtained the approximate stationary probability density of system response of quasi-integrable Hamiltonian systems with delayed feedback control by

* Corresponding author. Tel.: +86 57187952651.
E-mail address: xiaolingjin@zju.edu.cn (X.L. Jin).

0093-6413/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.mechrescom.2012.05.001

using the stochastic averaging method. Transient responses of time-delay MDOF systems are important due to the effects of time delay on system stability. Unfortunately, very few works on transient probability densities of time-delay MDOF systems have been considered.

In the present paper, transient probability densities of nonlinear MDOF systems with time delay subject to stochastic external and/or parametric excitations are investigated. The time-delay systems are appropriately replaced by the corresponding non-time-delay systems. By using the stochastic averaging procedure based on the generalized harmonic functions, FPK equation governing the transient probability densities for amplitude responses is obtained. Based on the similar procedures proposed by Spanos et al. (2007), the transient probability density is expressed as a multiple series expansion in terms of a set of properly state-dependent basis functions with time-dependent coefficients, which can be solved by a set of first-order equations by the Galerkin method. The transient probability densities for state-space response can also be obtained from that for amplitude response. Two examples are given to illustrate the accuracy and efficacy of the proposed procedure.

2. Model of MDOF systems with time delay

Consider a nonlinear MDOF system subject to stochastic external and/or parametric excitations. The equations of motion of the system are as follows:

$$\ddot{X}_i + \varepsilon c_{ij}(\mathbf{X}, \dot{\mathbf{X}})\dot{X}_j + g_i(X_i) + \varepsilon Z_i(X_{i\tau}, \dot{X}_{i\tau}) = \varepsilon^{1/2} f_{ik}(\mathbf{X}, \dot{\mathbf{X}}) W_k(t) \quad i, j = 1, \ldots, n; \quad k = 1, \ldots, m \tag{1}$$

where $\mathbf{X} = [X_1, \ldots, X_n]^{\mathrm{T}}$ and $\dot{\mathbf{X}} = [\dot{X}_1, \ldots, \dot{X}_n]^{\mathrm{T}}$ are generalized displacements and velocities, respectively, which are zeros as $t < 0$, ε a small positive parameter, $c_{ij}(\mathbf{X}, \dot{\mathbf{X}})$ the nonlinear damping coefficients, $g_i(X_i)$ the uncoupled stiffnesses, which are odd functions of the generalized displacements X_i, i.e., $g_i(-X_i) = -g_i(X_i)$, $\varepsilon Z_i(X_{i\tau}, \dot{X}_{i\tau})(X_{i\tau} = X_i(t-\tau), \dot{X}_{i\tau} = \dot{X}_i(t-\tau))$ the time-delay terms, which are delayed feedback control forces in control engineering and they vanish as $t - \tau < 0$, and usually chosen as polynomial of $X_{i\tau}$ and $\dot{X}_{i\tau}$, $\varepsilon^{1/2} f_{ik}(\mathbf{X}, \dot{\mathbf{X}})$ the magnitudes of external and/or parametric excitations, assumed as polynomial of $\mathbf{X}$ and $\dot{\mathbf{X}}$, and $W_k(t)$ the independent Gaussian white noises with intensities $2D_{kk}$. For simplicity, it is assumed that for each degree of freedom (also called subsystem) in system (1), only one term of external excitation is considered, which is described as $\varepsilon^{1/2} f_{il_i} W_{l_i}(t)$ $(l_i \in \{1, \ldots, m\})$ where f_{il_i} is a constant. The Einstein notation is used in $c_{ij}(\mathbf{X}, \dot{\mathbf{X}})\dot{X}_j$ and $f_{ik}(\mathbf{X}, \dot{\mathbf{X}})W_k(t)$ of Eq. (1). In the present paper, only the case of non-internal resonance is studied.

System (1) can be simplified to a MDOF Hamiltonian system as $\varepsilon = 0$, which has the form of

$$\ddot{x}_i + g_i(x_i) = 0 \quad i = 1, \ldots, n \tag{2}$$

The first integrals of system (2) is

$$H_i = \frac{\dot{x}_i^2}{2} + U_i(x_i) \quad i = 1, \ldots, n \tag{3}$$

in which $U_i(x_i) = \int_0^{x_i} g(u_i) du_i$ are the potential energies.

Supposing that the solutions to system (2) periodically surround the origin equilibrium point respectively, and they can be expressed as (Xu and Chung, 1994):

$$x_i(t) = a_i \cos\theta_i(t), \quad \dot{x}_i(t) = -a_i \nu_i(a_i, \theta_i) \cos\theta_i(t), \quad \theta_i(t) = \phi_i(t) + \varphi_i(t) \quad i = 1, \ldots, n \tag{4}$$

where a_i are the instantaneous amplitudes and

$$\nu_i(a_i, \theta_i) = \frac{d\phi_i}{dt} = \sqrt{\frac{2[U_i(a_i) - U(a_i \cos\theta_i)]}{a_i^2 \sin^2\theta_i}} \tag{5}$$

are the instantaneous frequencies, and the averaged frequencies can be obtained as:

$$\omega_{a_i}(a_i) = \frac{2\pi}{\int_0^{2\pi} \nu_i^{-1}(a_i, \theta_i) d\theta_i} \tag{6}$$

By using $H_i = U_i(a_i)$, the above averaged frequencies also have the expressions of

$$\omega_{H_i}(H_i) = \omega_{a_i}(a_i)|_{a_i = U_i^{-1}(H_i)} \tag{7}$$

in which $U_i^{-1}(H_i)$ are the inverse functions of $H_i = U_i(a_i)$.

When ε is small, the following approximation can be obtained (Liu and Zhu, 2007):

$$X_i(t-\tau) \approx X_i(t)\cos\ \omega_{a_i}\tau - \frac{\dot{X}_i(t)\sin\omega_{a_i}\tau}{\omega_{a_i}}, \quad \dot{X}_i(t-\tau) \approx \dot{X}_i(t)\cos\ \omega_{a_i}\tau + X_i(t)\omega_{a_i}\ \sin\omega_{a_i}\tau \tag{8}$$

Based on Eq. (8), the time-delay terms $Z_i(X_{i\tau}, \dot{X}_{i\tau})$ can be approximately replaced by $Z_i(X_{i\tau}, \dot{X}_{i\tau}) \approx Z_{1i}(X_i, \tau) + Z_{2i}(X_i, \dot{X}_i, \tau)\dot{X}_i$. Thus, system (1) can be rewritten as:

$$\ddot{X}_i + \varepsilon c_{ij}\dot{X}_j + \varepsilon Z_{2i}\dot{X}_i + g_i(X_i) + \varepsilon Z_{1i} = \varepsilon^{1/2} f_{ik}(\mathbf{X}, \dot{\mathbf{X}}) W_k(t) \quad i, j = 1, \ldots, n; \quad k = 1, \ldots, m \tag{9}$$

It should be pointed out that the systems considered in this paper are similar to those in Liu and Zhu (2007). In the present paper, the transient response is studied, while in Liu and Zhu (2007), only the stationary response is calculated, which is a special case of the present study.