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An investigation on primary resonance phenomena of elastic medium based single walled carbon nanotubes

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ABSTRACT

In this paper, the geometrically nonlinear free and forced oscillations of simply supported single walled carbon nanotubes (SWCNTs) are analytically investigated on the basis of the Euler–Bernoulli beam theory. The nonlinear frequencies of SWCNTs with initial lateral displacement are discussed. Equations have been solved using an exact method for free vibration and multiple times scales (MTS) method for forced vibration and some analytical relations have been obtained for natural frequency of oscillations. The numerical results reveal that the nonlinear free and forced vibration of nanotubes is effected significantly by both surrounding elastic medium and CNT aspect ratio.

1. Introduction

In recent years, miniaturized products have become important due to their ability to enhance the quality of many aspects of life. Carbon nanotubes (CNTs) are components of nanoscale dimensions that simultaneously present novel physical, mechanical and electrical properties. These properties have made them potentially useful for many applications in nanotechnology, electronics, optics and other fields of materials science. As a result, progressive research activities regarding CNTs have been ongoing in recent years. The use of CNTs in biological and biomedical applications creates hope for effective solutions to incurable illnesses, for instance. Sinha and Yeow (2005) have reported applications where CNTs can be used as diagnostic tools and devices and auxiliary tools in biopharmaceutics as well as implantable materials and devices.

There is a wide range of applications in which the vibrational characteristics of CNTs are significant. In applications such as oscillators, charge detectors, field emission devices, vibration sensors, and electromechanical resonators, oscillation frequencies are key properties. Hence, it is important to develop accurate theoretical models for evaluation of natural frequencies and mode shapes of CNTs. There are already exploratory studies on the continuum models for vibration of carbon nanotubes (CNTs) or similar micro or nanobeam like elements (Wang et al., 2006a,b; Rafiee and Nezamabadi, 2011; Shooshtari and Rafiee, 2011; Wang and Varadan, 2006; Lu et al., 2007). A review related to the importance and modeling of vibration behavior of various nanostructures can be found in Gibson's et al. (2007). In these works it has been suggested that nonlocal elasticity theory developed by Eringen (1983, 2002) should be used in the continuum models for accurate prediction of vibration behaviors. This is due to the scale effect of the nanostructures. Importance of accurate prediction of nanostructures' vibration characteristics has been discussed by Gibson et al. (2007). Kwon et al. (2005) presented a direct method for evaluating the natural frequencies and mode shapes of various CNTs. In that effort, the Tersoff–Brenner interatomic potential describing the interactions between carbon atoms was utilized for the development of the stiffness matrix while the atomic masses were used for the construction of the mass matrix. Liu et al. (2004) have developed an order *N*, atomic-scale finite element method that can handle discrete atoms and account for the multi-body interactions among atoms. The nature of this method enables the study of large scale problems.

Up until now, most of the investigations carried out on the vibration of CNTs have been restricted to the linear regime. These nanostructured materials can undergo large deformations within the elastic limit and their nonlinear analysis is clearly essential. Developing powerful predictive models to study various aspects of the vibrational behavior of CNTs is of high importance. Generally, owing to the complexities of nonlinear analysis, there has been no single overarching principle that governs the solution of nonlinear problems. There are, however, a number of general approaches that can be adopted for the solution of a certain class of nonlinear problems. Recently, Fu et al. (2006) studied the nonlinear

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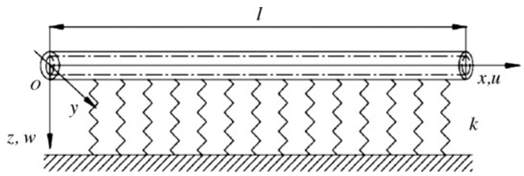

Fig. 1. Geometric representation of an embedded carbon nanotube.

vibrations of embedded nanotubes, with the inclusion of intertube radial displacements and the internal vdW forces, using the incremental harmonic balanced method (IHBM). They used the Euler–Bernoulli uncoupled beam theory on which the theoretical formulation is based. In that work, single-walled nanotubes (SWNTs) and double-walled nanotubes (DWNTs) were considered for the study. In a related work, Ansari and his co-workers (2010) investigated the nonlinear vibrations of single-, double- and triple-walled nanotubes (TWNTs) on the basis of the uncoupled Euler–Bernoulli equation using homotopy perturbation method (HPM). The present work can be regarded as an extension of the authors' previous work on nonlinear vibrational response of MWNTs using the variational iteration method (VIM). Joshi et al. (2010a) investigated dynamic analysis of carbon nanotube with surface deviation along its axis. The type of carbon nanotube used in their analysis is a single-walled carbon nanotube that is doubly clamped at a source and a drain and this type of nanotube is used to represent a single mode resonator. In another work done by Joshi and his coworkers (2010b), the simulation of the mechanical responses of individual carbon nanotubes treated as thin shells with thickness has been done using FEM.

Beyond the theoretical, molecular mechanics and continuum methods experimental techniques are also proposed in the literature in order to evaluate the oscillatory frequencies of CNTs. However, detection of mechanical vibrations of nanotubes is considered a very difficult task. Garcia-Sanchez et al. (Garcia-Sanchez et al., 2007) have recently presented a mechanical method for CNT resonator vibration detection that uses a novel scanning force microscopy method. Verification of their results was done via comparisons with results obtained using elastic beam theory and assuming that Young's modulus is equal to 1TPa. The comparison between experimental and theoretical methods pre-require the complete definition of all parameters such as the length of the vibrating nanotube, the nanotube type and other conditions that influence the vibrational behavior such as the slack phenomenon, nature of the support condition, environmental conditions and other influences.

To the best of the authors the primary resonance of SWCNTs has not been reported. In this paper, based on the continuum mechanics and an elastic beam model, the nonlinear free and forced frequency analysis of SWCNTs considering intertube radial displacement and the related internal degrees of freedom rested on elastic foundation is investigated.

2. Formulation

The system under consideration is a simply supported CNT of length l, mass density ρ, cross-sectional area A and cross-sectional moment I, embedded in an elastic medium as shown in Fig. 1. Assume that $w(x, t)$ is displacement corresponding to the vertical direction, in terms of the spacial coordinate x and the time variable t.

The nonlinear equation of motion of CNTs is given by

$$EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} = \left[\frac{EA}{2l}\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 dx\right]\frac{\partial^2 w}{\partial x^2} + q(x,t) \quad (1)$$

in which $m = \rho A$ is the CNT mass of per unit length and $q(x, t)$ is the interaction pressure per unit axial length between the outermost tube and the surrounding medium, which can be described by the Winkler-like model (Lanir and Fung, 1972; Hahn and Williams, 1984), and

$$q = -kw \quad (2)$$

where the negative sign indicates that the pressure p is opposite to the deflection of the outmost tube, and k is a constant determined by the material constants of the surrounding elastic medium. Substituting Eq. (2) into

Eq. (1), that gives

$$EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} + kw = \left[\frac{EA}{2l}\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 dx\right]\frac{\partial^2 w}{\partial x^2} \quad (3)$$

3. Free vibration

Assume that the nanotube is simply supported at the two ends. So, the unknown function $w(x, t)$ may be given as

$$w(x,t) = \zeta(t)\sin\frac{\pi x}{l} \quad (4)$$

It satisfies the boundary condition: $x = 0, l$: $w = 0$, $M = 0$, in which M is the stress couple. Defining the following quantities

$$\omega_k = \sqrt{\frac{k}{\rho A}}, \quad \omega_l = \frac{\pi^2}{l^2}\sqrt{\frac{EI}{\rho A}}$$
$$r = \sqrt{I/A}, \quad \chi = \frac{1}{4} \quad (5)$$

and substituting Eq. (4) into Eq. (3), the nonlinear differential equation for the time function $\zeta(t)$ can be obtained as follows:

$$\ddot{\zeta} + (\omega_l^2 + \omega_k^2)\zeta + \chi r^{-2}\omega_l^2\zeta^3 = 0. \quad (6)$$

By introducing the following quantities,

$$\eta = \frac{\zeta}{r}, \quad \tau = (\omega_l^2 + \omega_k^2)^{1/2} t \quad (7)$$

one may obtain the following nondimensional nonlinear differential equation of the system

$$\ddot{\eta} + \eta + \gamma\eta^3 = 0. \quad (8)$$

where $\gamma = \chi\omega_l^2/(\omega_l^2 + \omega_k^2)$.

The initial conditions considered in the current work are:

$$\zeta(l/2, 0) = \zeta_{\max}, \quad \frac{\partial\zeta(l/2,0)}{\partial t} = 0 \quad (9)$$

The dimensionless initial conditions given by Eq. (9) become

$$\eta(1/2, 0) = \eta_{\max}, \quad \frac{\partial\eta(1/2,0)}{\partial\tau} = 0 \quad (10)$$

It is worth noting that Eq. (8) is a classical Duffing-type equation which represents a nonlinear oscillator without damping. This equation may be solved via various methods, such as the method of harmonic balance, equivalent linearization, generalized averaging and multiple scales method (Nayfeh and Mook, 1979). By multiplying Eq. (8) by $\dot{\eta}$, integrating with respect to time and using Jacobi elliptic function, the corresponding nonlinear frequency for this nonlinear problem for each mode is defined by using the following equation (Lestari and Hanagud, 2001):

$$\omega_{nl} = \frac{\pi\sqrt{1 + \eta_{\max}^2\gamma}}{2\bar{K}} \quad (11)$$