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An elastic micropolar mixture theory for predicting elastic properties of cellular materials

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Abstract

An efficient modeling approach is established to predict the elastic response of cellular materials with distributions of cell geometries. The approach does not require complex and time-consuming computational techniques usually associated with modeling such materials. Unlike most current analytical techniques, the modeling approach directly accounts for the cellular material microstructure. The approach combines micropolar elasticity theory and elastic mixture theory to predict elastic response of cellular materials to a wide range of loading conditions. The modeling approach is applied to the two-dimensional balsa wood material. Predicted properties are in good agreement with experimentally determined properties, which emphasizes the model's potential to predict the elastic response of other cellular solids, such as open cell and closed cell foams.

Keywords: Couple stress theory; Gradient theory; Honeycombs; Cosserat elasticity; Lattice structures; Frames

1. Introduction

Cellular materials have a lattice architecture that in many cases results in high specific stiffness, specific strength, and good thermal insulation properties relative to many engineering materials. Cellular materials have been used in many structural engineering applications, including the core material in composite sandwich panels (Gibson and Ashby, 1999). The design and implementation of cellular materials relies on accurate and efficient models to relate the lattice microstructure to the bulk mechanical properties.

Cellular materials consist of a complex interconnected framework of either material struts only (open cell foams) or material struts and cell face membranes (closed cell foams) that yields a porous- or a closed-cellular material, respectively. For example, the open cell lattice of the polyurethane foam shown in Fig. 1 has pores and material struts with a range of cell sizes and shapes distrib-

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Nomenclature

$a_k^{(n)}$ acceleration vector component of the nth constituent
$a_{kl}^{(n)}$ material derivative of $\varepsilon_{kl}^{(n)}$
$A_{kl}^{(n)}, A_{klmn}^{(n)}$ material moduli of the nth constituent
$A^{(n)}$ cross-sectional area of the struts of the nth constituent
a_{kl} material derivative of ε_{kl}
$b_{kl}^{(n)}$ material derivative of $\gamma_{kl}^{(n)}$
$B_{kl}^{(n)}, B_{klmn}^{(n)}$ material moduli of the nth constituent
b_{kl} material derivative of γ_{kl}
C_0 constant relating temperature and free energy in the natural state
$C_{klmn}^{(n)}$ material moduli of the nth constituent
D set of all dependent variables
$E_{\text{strut}}^{(n)}$ Young's modulus of the strut material of the nth constituent
$E^{(n)}$ Young's modulus of the equivalent continuum of the nth constituent
E Young's modulus of the mixture
$f_k^{(n)}$ body force density vector components of the nth constituent
$f^{(n)}$ volume fraction of the nth constituent
$G_{\text{strut}}^{(n)}$ shear modulus of the material of the struts of the nth constituent
$G^{(n)}$ shear modulus of the equivalent continuum of the nth constituent
G shear modulus of the mixture
$h^{(n)}$ internal energy source density of the nth constituent
h internal energy source density of the mixture
I set of all independent variables
$I^{(n)}$ moment of inertia of the struts of the nth constituent
$j^{(n)}$ microinertia density of the nth constituent
J set of all thermodynamic fluxes
K classical Fourier constant
$l_i^{(n)}$ body couple density vector components of the nth constituent
$l^{(n)}$ length of the struts of the nth constituent
$m_{kl}^{(n)}$ couple stress tensor components of the nth constituent
$\hat{m}_k^{(n)}$ internal couple vector components exerted onto nth constituent by the other constituent
m_{kl} couple stress tensor components of the mixture
$\hat{p}_k^{(n)}$ internal force density vector components exerted onto nth constituent by the other constituent
$q_k^{(n)}$ heat flux vector components of the nth constituent
q_k heat flux vector components of the mixture
$Q^{(n)}$ grid structural parameter
$\mathscr{R}^{(n)}$ region occupied by the nth constituent in the mixture
$R^{(n)}$ Grid structural parameter
S_0 free energy in the natural state
$s^{(n)}$ cross-sectional thickness/equivalent continuum thickness of the nth constituent
$S^{(n)}$ grid structural parameter
t time at the end of constituent motion
$t_{kl}^{(n)}$ stress tensor components of the nth constituent
T change in temperature from ambient temperature
T_0 ambient temperature
$T^{(n)}$ grid structural parameter
t_{kl} stress tensor components of the mixture