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Axial vibration analysis of nanorods (carbon nanotubes) embedded in an elastic medium using nonlocal elasticity

Metin Aydogdu*

Department of Mechanical Engineering, Trakya University, 22180 Edirne, Turkey

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ABSTRACT

The axial vibration of single walled carbon nanotube embedded in an elastic medium is studied using nonlocal elasticity theory. The nonlocal constitutive equations of Eringen are used in the formulations. The effect of various parameters like stiffness of elastic medium, boundary conditions and nonlocal parameters on the axial vibration of nanorods is discussed. It is obtained that, the axial vibration frequencies of the embedded nanorods are highly over estimated by the classical continuum rod model which ignores the effect of small length scale.

1. Introduction

In the last two decades, nano-scale engineering materials and their technological applications have taken great interest after invention of carbon nanotubes (CNTs) by Iijima (1991). Previous studies related with CNTs (Dai et al., 1996; Falvo et al., 1997; Kim and Lieber, 1999; Kong et al., 2000; Bachtold et al., 2001; Dharap et al., 2004) have shown that CNTs have good electrical properties and high mechanical strength so they can be used for nanoelectronics, nanodevices, nanosensors and nanocomposites.

Since, molecular dynamic simulations are restricted to small scale systems and to short time intervals, continuum mechanics models were generally preferred to investigate elastic response of CNTs in the previous studies. Initially, the classical continuum mechanics models were directly applied to study bending, buckling and vibration of CNTs. The classical Euler–Bernoulli, Timoshenko and higher order shear deformation beam and shell models were applied to study wave propagation, bending, buckling and vibration of CNTs (Ru, 2000; Yoon et al., 2005; Wang and Varadan, 2006; Aydogdu, 2008a,b). After these applications of the classical continuum mechanics, its size independence was investigated. Sun and Zhang (2003) studied the limitations of continuum models in the nanometer length scale. They found that material properties dependent on the length of plate structure. These results indicate that discrete material structure at the nanoscale cannot be homogenized into a continuum. At this point, the nonlocal elastic continuum models were considered in the analysis of nanostructures.

The nonlocal elasticity was first considered by Eringen (1976,1983). He assumed that the stress at a reference point is a functional of the strain field at every point of the continuum. Peddieson et al. (2003) have used nonlocal Euler–Bernoulli model for static analysis of nano beams and they concluded that the nonlocal mechanics can be useful at nano length scale. Sudak (2003) applied the nonlocal elasticity for column buckling. Static analysis of micro and nano structures was studied by Wang and Liew (2007) using the nonlocal Euler–Bernoulli and Timoshenko beam theories. Wave propagation in CNTs is investigated by Wang (2005), Lu et al. (2007), Narendar and Gopalakrishnan (2010), and Narendar (2011). Vibration of CNTs, nano beams and rods were also studied in the previous studies using the nonlocal elasticity (Ece and Aydogdu, 2007; Aydogdu and Ece, 2007; Aydogdu, 2009a,b; Karaoglu and Aydogdu, 2010; Filiz and Aydogdu, 2010; Şimşek, 2010; Demir et al., 2010; Arash and Wang, 2012).

The nonlocal continuum models and molecular dynamic simulations were compared for wave propagation in SWCNTs and double walled carbon nanotubes (DWCNTs) (Hu et al., 2008). Good agreement was observed between molecular dynamic simulations and nonlocal continuum modeling. Recently three dimensional behavior of CNT was investigated by some researchers (Yuzhou and Liew, 2008; Gupta and Batra, 2008; Silvestre, 2008).

An advantage of CNTs is that due to their high stiffness, they are promising candidate as reinforced fiber embedded in composites. Recently, considerable attention has turned to mechanical behavior of single walled and multiwalled carbon nanotubes embedded in a polymer or metal matrix. Transverse vibration of carbon

* Tel.: +90 284 2261217x3015; fax: +90 284 2126067.
E-mail addresses: metina@trakya.edu.tr, metaydogdu@gmail.com

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nanotubes embedded in an elastic medium was investigated by many researchers using the local and nonlocal continuum models (Yoon et al., 2003; Aydogdu, 2008a,b; Murmu and Pradhan, 2009; Ke et al., 2009; Kiani, 2010; Ansari et al., 2011; Ansari and Hemmatnezhad, 2011).

The axial vibration of carbon nanotubes with uniform and nonuniform cross-sections was investigated using the nonlocal continuum models (Aydogdu, 2009a,b; Danesh et al., 2012). To the best of the author knowledge, the axial vibration of the nanorods embedded in an elastic medium was not considered in the previous studies. So, the main objective of this study is to fill this gap in the literature.

In this study, axial vibration of SWCNT embedded in an elastic medium is studied using the nonlocal elasticity theory. The effect of various parameters like stiffness of elastic medium, boundary conditions, geometrical properties of nanotubes and nonlocal parameters on the axial vibration of nanorods is investigated.

2. The nonlocal elasticity model for carbon nanotubes

Consider a SWCNT of length L and diameter d. The nonlocal constitutive relations can be given as (Lu et al., 2007; Aydogdu, 2009a,b):

$$(1-\mu\nabla^2)\tau_{kl} = \lambda\varepsilon_{rr}\delta_{kl} + 2G\varepsilon_{kl} \tag{1}$$

where τ_{kl} is the nonlocal stress tensor, ε_{kl} is the strain tensor, λ and G are the Lame constants, $\mu = (e_0a)^2$ is *called the nonlocal parameter*, a is an internal characteristic length and e_0 is a constant. Choice of e_0a (in dimension of length) is crucial to ensure the validity of nonlocal models. This parameter was determined by matching the dispersion curves based on the atomic models (Eringen, 1983). For a specific material, the corresponding nonlocal parameter can be estimated by fitting the results of atomic lattice dynamic and experiment. A conservative estimate of the scale coefficient $e_0a < 2.0$ nm for a SWCNT was proposed (Wang and Wang, 2007). In this study, $0 \le (e_0a)^2 \le 2$ is chosen in order to investigate nonlocality effects. In the following sections parameter μ is used instead of $(e_0a)^2$.

For the axial vibration of uniform CNT, Eq. (1) can be written in the following one dimensional form:

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right)\tau_{xx} = E\varepsilon \tag{2}$$

where E is the modulus of elasticity. The equation of motion for the axial vibration can be obtained as

$$\frac{\partial N}{\partial x} + f = m\frac{\partial^2 u}{\partial t^2} \tag{3}$$

where $u(x, y)$ is the axial displacement, m is the mass per unit length, f is the distributed axial force acting on the rod and N is the axial force per unit length defined by

$$N = \int_A \sigma_{xx} dA \tag{4}$$

where A is the cross-sectional area of the CNT and σ_{xx} is the local stress component in the x direction. Integrating Eq. (2) with respect to area gives the following relation:

$$N - \mu\frac{\partial^2 N}{\partial x^2} = N^L \tag{5}$$

where $N = \int_A \tau_{xx} dA$ and N^L denote axial force per unit length for the nonlocal elasticity and local elasticity respectively. Using Eqs. (3)–(5) the following equation of motion for free axial vibration of nanorod can be found in terms of displacement:

$$EA\frac{\partial^2 u}{\partial x^2} + f - \mu\frac{\partial^2 f}{\partial x^2} = \left(1-\mu\frac{\partial^2}{\partial x^2}\right)m\frac{\partial^2 u}{\partial t^2} \tag{6}$$

Eq. (6) is the consistent fundamental equation of the nonlocal rod model for the axial vibration of CNT. This equation is reduced to the equation of the classical rod model if the nonlocal parameter μ is identically zero.

2.1. Governing equations for SWCNTs embedded in an elastic medium

In order to increase the strength of composites, CNTs are commonly embedded in an elastic medium, and the surrounding elastic medium has strong effect on mechanical behavior of CNTs. To analyze axial vibration of embedded CNTs, a model is proposed in the present study. Now, consider a typical SWCNT embedded in an elastic medium (Fig. 1).

In this study, axial force due to elastic medium is assumed in the following form:

$$f = -ku \tag{7}$$

where k is the stiffness of the elastic medium. Inserting Eq. (7) into Eq. (6) leads to following equation of motion for a nanorod embedded in elastic medium.

$$EA\frac{\partial^2 u}{\partial x^2} - ku + \mu k\frac{\partial^2 u}{\partial x^2} = \left(1-\mu\frac{\partial^2}{\partial x^2}\right)m\frac{\partial^2 u}{\partial t^2} \tag{8}$$

when $k = 0$, Eq. (8) is reduced to the nonlocal rod equation of motion without an elastic medium. To study the axial vibration of a nanorod embedded in an elastic medium Eq. (8) should be solved for given boundary conditions. Assuming harmonic vibration, $u(x,t)$ can be written in the following form:

$$u(x,t) = U(x)e^{j\omega t} \tag{9}$$

where ω is the circular frequency and $j^2 = -1$. Introducing Eq. (9) into Eq. (8) gives following dimensionless equation of motion

$$\frac{d^2U}{dx^2} + \beta^2 U = 0 \tag{10}$$

where related coefficients are defined as

$$\beta^2 = \frac{\Omega^2 - \bar{K}}{1-(\mu/L^2)\Omega^2 + (\mu/L^2)\bar{K}}, \tag{11a}$$

$$\Omega^2 = \frac{m\omega^2L^2}{EA}, \bar{K} = \frac{kL^2}{EA} \tag{11b}$$

where Ω and $\bar{K}$ are the non-dimensional frequency parameter and stiffness parameter respectively. The general solutions of Eq. (10) can be written in the following form:

$$U(x) = C_1\cos(\beta x) + C_2\sin(\beta x) \tag{12}$$

where C_i ($i = 1,2$) are the undetermined coefficients. To determine the frequency parameter and mode shapes of the nano rod boundary conditions should be given. In this study, clamped-clamped (C-C) and clamped-free (C-F) boundary conditions are studied using following relations:

$$\begin{aligned} &\text{C}: \quad u = 0, \\ &\text{F}: \quad N = EA\frac{\partial u}{\partial x} + (e_0a)^2 m\frac{\partial^3 u}{\partial x\partial t^2} = 0 \end{aligned} \tag{13}$$