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Static bending behaviors of nanoplate embedded in elastic matrix with small scale effects

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ABSTRACT

In this paper, the bending behaviors of the nanoplate with small scale effects are investigated by the nonlocal continuum theory. The governing equations for the nonlocal Mindlin and Kirchhoff plate models are derived. The expressions of the bending displacement are presented analytically. The difference between the two models is discussed and bending properties of the nanoplate are illustrated. It can be observed that the small scale effects are obvious for bending properties of the nanoplate. The half wave numbers, width ratios and elastic matrix properties also have significant influence on bending behaviors.

1. Introduction

Since the pioneer work on carbon nanotubes by Iijima (1991), extensive research interests on nano structures have been conducted (Baughman et al., 2002; Szabados et al., 2006; Narendar et al., 2011). With the superior characteristics, many potential applications can be expected such as the atomic-force microscope, field emitters and nanoscale electronic devices in nano electromechanical systems (NEMS). Among various excellent properties of nano stuctures, the mechanical characteristics are important for the design and analysis of the nanoscale devices and have received lots of attention (Thostenson et al., 2001; Lau et al., 2006; Gibson et al., 2007; Chong, 2008; Scarpa et al., 2009).

Since it is very difficult to perform the experiment at the nanoscale and time consuming to do the molecular dynamics (MD) simulation, many researches tune to apply the elastic continuum models to investigate the mechanical characteristics of nano structures (Wang and Cai, 2006; Zhang et al., 2008; Lee et al., 2009; Natsuki et al., 2010; Soltani et al., 2010). In recent years, in order to consider the small scale effects, the nonlocal elastic theory presented by Eringen (1972, 1983) has presented the reliable and proper results to shown the mechanical behaviors of nano structures (Peddieson et al., 2003; Wang and Varadan, 2006; Lu et al., 2007a; Li et al., 2008; Ke et al., 2009).

The nanoplate is a typical structure of nanoscale systems, which can be deformed into the nanotube and made as the MEMS/NEMS component. However, different from the researching status for the nanotube, only several works have been reported on the nanoplate. Kitipornchai et al. studied the vibration characteristics of multilayered grapheme sheets (Kitipornchair et al., 2005). Duan and Wang investigated the axisymmetric bending of circular plates with the micro/nanoscale (Duan and Wang, 2007). Murmu and Pradhan has presented the vibration and buckling behaviors of the nanoplate (Murmu and Pradhan, 2009a,b,c). In our recent work (Wang et al., 2010a,b), the propagation characteristics of the longitudinal and flexural waves in the nanoplate are investigated. It has shown that the small scale effects are obvious for the mechanical characteristics of the nanoplate.

The analytical method for the nanoplate is mainly based on the Kirchhoff plate theory and usually the effect of the shear deformation is not considered. In the present work, the bending behaviors are studied with the nonlocal Mindlin plate theory. The results are presented as the displacement ratio of the nonlocal Mindlin to nonlocal Kirchhoff plate models. Some influences (e.g. the scale coefficient, half wave number and elastic matrix properties etc.) on the bending properties are discussed.

2. Governing equations

Fig. 1 shows the nanoplate embedded in the elastic matrix. The widths along the x and y direction is l_a and l_b, respectively. The thickness is h and the external load is q. According to the nonlocal continuum theory (Eringen, 1972, 1983), which accounts for the small scale effects by assuming the stress at a reference point as a

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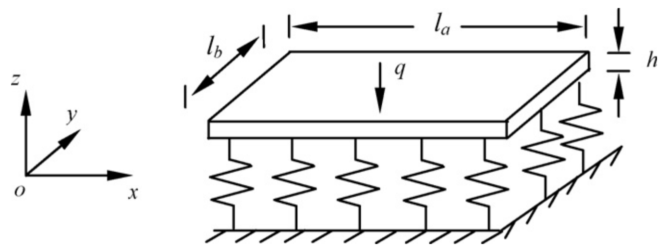

Fig. 1. Nanoplate embedded in elastic matrix with external loads.

function of the strain at every point in the body, the constitutive relation can be presented as the following integral form:

$$\sigma_{kl,\,k} - \rho\,\ddot{u}_l = 0, \tag{1a}$$

$$\sigma_{kl}(\mathbf{x}) = \int_V \alpha(\mathbf{x}, \mathbf{x}')\tau_{kl}(\mathbf{x}')dV(\mathbf{x}'), \tag{1b}$$

$$\varepsilon_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}), \tag{1c}$$

where σ_{kl} is the nonlocal stress tensor, ε_{kl} the strain tensor, ρ the mass density, u_l the displacement vector, $\tau_{kl}(\mathbf{x}')$ the classical (i.e. local) stress tensor, $\alpha(\mathbf{x}, \mathbf{x}')$ the kernel function which describes the influence of the strains at various location $\mathbf{x}'$ on the stress at a given location $\mathbf{x}$ and V the entire body considered.

It can be observed that the spatial integrals are involved in the nonlocal constitutive relation, which results in the difficulty for the problem. However, these integral equations can be reduced to the partial differential forms and the nonlocal constitutive relation can be employed conveniently. Then, the Hook's law can be expressed as

$$\sigma_x - (e_0a)^2\nabla^2\sigma_x = \frac{E}{1-\upsilon^2}(\varepsilon_x + \upsilon\varepsilon_y), \tag{2a}$$

$$\sigma_y - (e_0a)^2\nabla^2\sigma_y = \frac{E}{1-\upsilon^2}(\varepsilon_y + \upsilon\varepsilon_x), \tag{2b}$$

$$\tau_{yz} - (e_0a)^2\nabla^2\tau_{yz} = \frac{E}{1+\upsilon}\varepsilon_{yz}, \tag{2c}$$

$$\tau_{xz} - (e_0a)^2\nabla^2\tau_{xz} = \frac{E}{1+\upsilon}\varepsilon_{xz}, \tag{2d}$$

$$\tau_{xy} - (e_0a)^2\nabla^2\tau_{xy} = \frac{E}{1+\upsilon}\varepsilon_{xy}, \tag{2e}$$

where E the Young's modulus, υ the Poisson's ratio, e_0 the constant appropriate to each material and a the internal characteristic length (e.g. the length of C–C bond, the lattice spacing and the granular distance) and e_0a means the scale coefficient which denotes the small scale effect on the mechanical characteristics. If $e_0a = 0$, this relation will be reduced to the classical local model.

With the effects of the transverse shear and rotary inertia, the displacements can be expressed as

$$u = u(x,y,t) + z\psi_x(x,y,t), \quad v = v(x,y,t) + z\psi_y(x,y,t),$$
$$w = w(x,y,t), \tag{3}$$

where ψ_x and ψ_y the local rotations for the x and y directions, respectively.

Furthermore, the strain can be expressed as

$$\varepsilon_x = \frac{\partial u}{\partial x} + z\frac{\partial \psi_x}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y} + z\frac{\partial \psi_y}{\partial y}, \quad \varepsilon_z = 0, \tag{4a}$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \psi_x\right), \quad \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \psi_y\right), \tag{4b}$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + z\frac{\partial \psi_x}{\partial y} + z\frac{\partial \psi_y}{\partial x}\right). \tag{4c}$$

The bending moments and shear forces are

$$M_x = \int_{-h/2}^{h/2}\sigma_x z\,dz, \quad M_y = \int_{-h/2}^{h/2}\sigma_y z\,dz,$$
$$M_{xy} = \int_{-h/2}^{h/2}\tau_{xy} z\,dz, \tag{5a}$$

$$S_x = \int_{-h/2}^{h/2}\tau_{xz}\,dz, \quad S_y = \int_{-h/2}^{h/2}\tau_{yz}dz. \tag{5b}$$

According to Eqs. (2a)–(2e), (5a) and (5b), we can derive the following relation:

$$M_x - (e_0a)^2\nabla^2 M_x = D\left(\frac{\partial \psi_x}{\partial x} + \upsilon\frac{\partial \psi_y}{\partial y}\right), \tag{6a}$$

$$M_y - (e_0a)^2\nabla^2 M_y = D\left(\frac{\partial \psi_y}{\partial y} + \upsilon\frac{\partial \psi_x}{\partial x}\right), \tag{6b}$$

$$M_{xy} - (e_0a)^2\nabla^2 M_{xy} = \frac{1}{2}D(1-\upsilon)\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right), \tag{6c}$$

$$S_x - (e_0a)^2\nabla^2 S_x = \kappa\,G\,h\left(\frac{\partial w}{\partial x} + \psi_x\right), \tag{6d}$$

$$S_y - (e_0a)^2\nabla^2 S_y = \kappa\,G\,h\left(\frac{\partial w}{\partial y} + \psi_y\right), \tag{6e}$$

where G is the shear modulus, $D = Eh^3/12(1-\upsilon^2)$ the bending stiffness and κ the shear correction factor.

The governing equations with the elastic matrix and the external load are (Murmu and Pradhan, 2009a; Reddy, 1997; Lu et al., 2007b; Pradhan and Phadikar, 2009; Achenbach, 1973)

$$\frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y} + q = k_w w - G_b\nabla^2 w, \tag{7a}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - S_x = 0, \tag{7b}$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - S_y = 0, \tag{7c}$$

where k_w the Winkler foundation modulus, G_b the stiffness of the shearing layer.

Based on Eqs. (6a)–(6e) and Eqs. (7a)–(7c), the governing equations of the nanoplate with the nonlocal Mindlin plate model can be derived as

$$\begin{aligned}&[1-(e_0a)^2\nabla^2]k_w w - G_b[1-(e_0a)^2\nabla^2]\,\nabla^2 w\\ &-\kappa\,Gh\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y}\right) = [1-(e_0a)^2\nabla^2]q\end{aligned}, \tag{8a}$$

$$D\left[\frac{\partial^2 \psi_x}{\partial x^2} + \frac{1}{2}(1-\upsilon)\frac{\partial^2 \psi_x}{\partial y^2} + \frac{1}{2}(1+\upsilon)\frac{\partial^2 \psi_y}{\partial x\partial y}\right] - \kappa\,Gh\left(\frac{\partial w}{\partial x} + \psi_x\right) = 0, \tag{8b}$$

$$D\left[\frac{\partial^2 \psi_y}{\partial y^2} + \frac{1}{2}(1-\upsilon)\frac{\partial^2 \psi_y}{\partial x^2} + \frac{1}{2}(1+\upsilon)\frac{\partial^2 \psi_x}{\partial x\partial y}\right] - \kappa\,Gh\left(\frac{\partial w}{\partial y} + \psi_y\right) = 0. \tag{8c}$$