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Contact mechanics of two deformable elastic solids with graded coatings

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Abstract

The contact problem for two deformable solids with FGM coatings is considered. The problem is considered under the assumption of plane strain, Coulomb friction and linear nonhomogeneous elasticity. The variation of the elastic modulus in the FGM coatings are exponential. It is assumed that the contact area is small compared to radii of curvatures of the cylinders. Thus the standard Hertzian assumption may be used. After solving the half space problem the Green's functions necessary for deriving the integral equations of the contact problem are obtained. The resulting integral equations are solved to obtain a series of analytical benchmark results for examining the influence of such factors as material inhomogeneity constants, the coefficient of friction, and various length parameters on the critical stresses that may have a bearing on the fatigue and fracture of the components with FGM coatings. The results regarding the contact stresses, the in-plane component of the surface stress and the contact length versus the applied load relationship are presented.

Keywords: Contact mechanics; Friction; Functionally graded materials; Hertzian contact

1. Introduction

A common mode of failure in many structural components is fatigue and fracture caused by contact stresses in areas of high stress concentrations. The proper preparation of surfaces, therefore, in load transfer components such as ball and journal bearings, gears, machine tools, cams, and abradable seals (in high performance stationary gas turbines) is a very important part of the related design and manufacturing processes. In failure analysis the underlying physical problem is initiation and propagation of surface cracks generated by the sliding contact in the presence of friction, which generally leads to fretting fatigue (Hills et al., 1993). Experimentally it was indeed observed that "herring bone" type surface cracking is formed in glass substrates loaded by a sliding spherical steel indenter (Lawn, 1993; Suresh et al., 1999; see also Bower and Fleck, 1994 for sliding cylindrical indenter).

The techniques used to prepare surfaces against contact damage include shot peening, case hardening, and more recently, thin film coatings. Largely due to the mismatch of thermomechanical

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properties of homogeneous substrates and coatings, the coating technique have certain structural shortcomings (Oliveria and Bower, 1996). A relatively new concept in material design introduced to overcome some of these shortcomings has been smoothly grading the thermomechanical properties of the coating by continuously varying the through-thickness composition of the surface layer which essentially is a multi-phase composite (Suresh and Mortensen, 1998; Miyamoto et al., 1999; Van der Biest et al., 2005). Such materials designed and processed for specific application have come to be known as the functionally graded materials (FGMs) (Yamanouchi et al., 1990). Used as coatings and interfacial zones they tend to reduce residual and thermal stresses resulting from the material property mismatch, increase the bonding strength, improve surface properties, and provide protection against severe thermal and chemical environments.

The technical literature in contact mechanics is very extensive. A thorough description and review of the problems involving homogeneous materials may be found in Gladwell (1980), Johnson (1985) and Hills et al. (1993). Studies in contact mechanics on the graded materials, on the other hand, are rather limited and carried out mostly by Suresh and his co-workers (Giannakopoulos and Suresh, 1997; Suresh et al., 1997; Suresh et al., 1999; see also Booker et al., 1985). The surface cracking of a graded medium loaded by a sliding rigid stamp was studied by Dag and Erdogan (2002). The contact mechanics of graded coatings bonded to homogeneous substrates and loaded by rigid stamps with rectangular and triangular profiles was considered by Guler and Erdogan (2004).

The problem considered in this study is described in Fig. 1. It is assumed that the contacting solids locally have shallow curvatures, that is the size $(a+b)$ of the contact zone is "small" compared to radii R_1 and R_2. Thus, in formulating the problem one may make the standard Hertzian assumption to the effect that the Green's functions for the concentrated surface tractions in a cylindrical medium may be approximated by that of a half plane.

The contacting solids consist of dissimilar homogeneous materials coated by graded elastic layers of known thickness. Locally the solids will be represented by circular cylinders with positive/negative (Fig. 1a) or positive/positive (Fig. 1b) curvatures. The problem will be considered with or without friction. The main calculated quantities will be the contact stresses and the load versus the contact zone size curves. Specifically the objective is the examination of the influence of friction coefficient and material inhomogeneity parameters on the peak surface stresses.

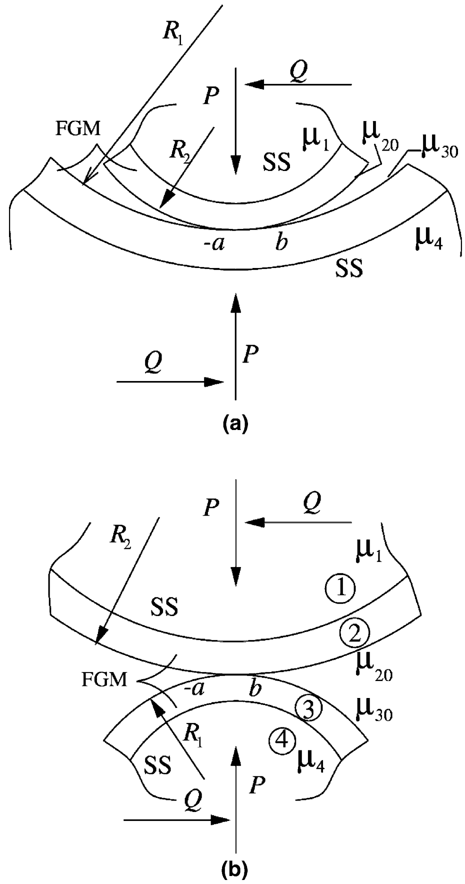

Fig. 1. Contact problems for load transfer components with FGM coatings.

In these problems it will be assumed that a coating layer of thickness h_2 or h_3 is perfectly bonded to the homogeneous substrate. The coatings are nonhomogeneous and the nonhomogeneity is assumed to be in the thickness direction only and may be approximated by $\mu_3(y)=\mu_{30}e^{\gamma_3 y}$, and $\mu_2(y)=\mu_{20}e^{\gamma_2 y}$, μ_{30} and μ_{20} being the shear moduli of the FGM coatings on the surface, $y=0$. The Poisson's ratio is assumed to be constant for both materials.

The result of this study may be applicable to a great variety of structural components such as connecting-rods, bolted connections, shrink fits, rolling mills, turbine blade roots, ball and roller bearings, foundations, pavements in roads and runways, and other structures consisting of layered media.