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On the constitutive relations and energy potentials of linear thermo-magneto-electro-elasticity

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ABSTRACT

The sixteen different types of constitutive relations of linear thermo-magneto-electro-elasticity derived following the analytical formulation of solids thermodynamics are presented. The thermodynamic potentials from which they are derived are also presented.

1. Introduction

Because of their applications in sensing and actuation, the development of materials exhibiting couplings between elastic, electric, magnetic and thermal fields have become of significant interest. Such coupled effects are stated together via the constitutive relations of thermo-magneto-electro-elasticity (TMEE), which can be understood as the link between Newton's equation of motion, Maxwell's equations of electromagnetism, and the heat equation. Considering only the linear framework, several studies involving such materials have been carried out. For instance, Aboudi (2000) applied micromechanical homogenization to fibrous and laminate composites with TMEE constituents in comparison with his generalized method of cells and the Mori–Tanaka method; Benveniste (1995) studied the magneto-electric effect of fibrous composites with piezoelectric and piezomagnetic phases taking thermal effects into account and derived connections between its effective properties; Chen et al. (2002) derived a micromechanical model for the effective properties of a layered composite with piezoelectric and piezomagnetic components using TMEE constitutive relations; Chen and Lee (2003) analyzed the bending of inhomogeneous transversely isotropic TMEE plates with simplified constitutive relations; Chen et al. (2004) presented a general solution for transversely isotropic TMEE materials employed to generalize the potential theory method applied to a crack subjected to mechanical, electric, and magnetic forces and temperature load; Li (2003) derived uniqueness and reciprocity theorems for TMEE media without making restrictions on the positive definiteness of the elastic moduli; in Li and Dunn (1998), a micromechanical approach for the average fields and effective properties of TMEE fibrous and layered composites is developed using the Mori–Tanaka method; Tan and Tong (2002) presented two micromechanical models to investigate the TMEE properties of piezoelectric–magnetic fibrous composites in comparison with the Mori–Tanaka method;

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Zheng and Chen (1999) extended results in plane elasticity to generalized plane deformation of TMEE materials including the correspondences between various physical contexts, invariance of stresses under a change in elastic compliance, and the reduced dependence of effective elastic compliance upon the material constants. However, except Li (2003) and Zheng and Chen (1999), the investigations mentioned above did not consider the constitutive relation between entropy and temperature as their main interest was the influence or apparition of the magneto-electric effect. Some of them spoke of magneto-electro-elasticity even though thermal effects were explicitly considered. Moreover, all the mentioned works were carried out with the same choice of independent variables, which in this paper is referred to as type-9 formulation (one of the sixteen possibilities). For such reason, this communication is devoted to present all the constitutive relations together with the corresponding energy potentials. Following the analytical thermodynamics of solids, such constitutive relations were obtained from the equations of state with physical properties defined as various partial derivatives of certain thermodynamic (energy) potentials having their own sets of natural independent variables in which the system is fully described (see Soh and Liu, 2005 for a full description in magneto-electro-elasticity). All through the paper, the summation convention over repeated lowercase Latin subscripts, which take values on the set {1,2,3}, is adopted.

2. Thermodynamic potentials and constitutive relations of linear TMEE media

It is known from the first and second laws of thermodynamics that, for quasi-static infinitesimal reversible processes, the small increment dU of the internal energy U of a system subject to mechanical, electric, magnetic and thermal influences from its surroundings, is given by

$$dU = \sigma_{kl} d\varepsilon_{kl} + E_n dD_n + H_q dB_q + TdS \tag{1}$$

Eq. (1) means that ε_{kl}, D_n, B_q and S (strain, electric displacement, magnetic induction, and entropy increment) are the natural independent variables of U, while σ_{kl}, E_n, H_q and T (stress, electric field, magnetic field, and temperature increment) are the dependent variables. As Eq. (1) expresses dU in terms of the natural variables (so it is a perfect differential) by identifying it with the formal differential of U, the equations of state are derived as equalities between the dependent variables (with some sign) and the first-order partial derivatives of U. The second-order partial derivatives of U defining the physical properties are derived from the equations of state as different first-order partial derivatives of the dependent variables arising from their total differentials. The other fifteen formulations (for which the names of the potentials are omitted here as they are well-established only in a few cases) are derived in a similar fashion after a suitable Legendre transformation is used to prevent information losses. In what follows, even-number equations state the particular forms of the potentials as a Legendre transformation of U (first equalities), as a formal energy potential of a linear system (second equalities), and as an explicit energy potential of a linear system (third equalities); and odd-number Eqs. show the equations of state (first equalities) from which the constitutive relations (second equalities) are derived (see the Appendix for a full derivation of the type-1 formulation).

Type 1: Formulation in σ_{ij}, E_m, H_p, T

$$\begin{aligned}2U_1 &= 2(U - \varepsilon_{kl}\sigma_{kl} - D_nE_n - B_qH_q - ST) = -\varepsilon_{kl}\sigma_{kl} - D_nE_n - B_qH_q - ST\\ &= -S^{E,H,T}_{ijkl}\sigma_{ij}\sigma_{kl} - \kappa^{\sigma,H,T}_{mn}E_mE_n - \mu^{\sigma,E,T}_{pq}H_pH_q - c^{\sigma,E,H}T^2 - 2d^{H,T}_{mij}\sigma_{ij}E_m - 2q^{E,T}_{pij}\sigma_{ij}H_p - 2\alpha^{E,H}_{ij}\sigma_{ij}T - 2m^{\sigma,T}_{pm}E_mH_p\\ &\quad - 2p^{\sigma,H}_mE_mT - 2t^{\sigma,E}_pH_pT\end{aligned} \tag{2}$$

$$\begin{cases}\varepsilon_{kl} = -(\partial U_1/\partial\sigma_{kl})_{E,H,T} = S^{E,H,T}_{ijkl}\sigma_{ij} + d^{H,T}_{mkl}E_m + q^{E,T}_{pkl}H_p + \alpha^{E,H}_{kl}T\\ D_n = -(\partial U_1/\partial E_n)_{\sigma,H,T} = d^{H,T}_{nij}\sigma_{ij} + \kappa^{\sigma,H,T}_{mn}E_m + m^{\sigma,T}_{pn}H_p + p^{\sigma,H}_nT\\ B_q = -(\partial U_1/\partial H_q)_{\sigma,E,T} = q^{E,T}_{qij}\sigma_{ij} + m^{\sigma,T}_{qm}E_m + \mu^{\sigma,E,T}_{pq}H_p + t^{\sigma,E}_qT\\ S = -(\partial U_1/\partial T)_{\sigma,E,H} = \alpha^{E,H}_{ij}\sigma_{ij} + p^{\sigma,H}_mE_m + t^{\sigma,E}_pH_p + c^{\sigma,E,H}T\end{cases} \tag{3}$$

Type 2: Formulation in σ_{ij}, E_m, H_p, S

$$\begin{aligned}2U_2 &= 2(U - \varepsilon_{kl}\sigma_{kl} - D_nE_n - B_qH_q) = -\varepsilon_{kl}\sigma_{kl} - D_nE_n - B_qH_q + ST\\ &= -S^{E,H,S}_{ijkl}\sigma_{ij}\sigma_{kl} - \kappa^{\sigma,H,S}_{mn}E_mE_n - \mu^{\sigma,E,S}_{pq}H_pH_q + \varsigma^{\sigma,E,H}S^2 - 2d^{H,S}_{mij}\sigma_{ij}E_m - 2q^{E,S}_{pij}\sigma_{ij}H_p - 2A^{E,H}_{ij}\sigma_{ij}T - 2m^{\sigma,S}_{pm}E_mH_p\\ &\quad - 2P^{\sigma,H}_mE_mT - 2\theta^{\sigma,E}_pH_pT\end{aligned} \tag{4}$$

$$\begin{cases}\varepsilon_{kl} = -(\partial U_2/\partial\sigma_{kl})_{E,H,S} = S^{E,H,S}_{ijkl}\sigma_{ij} + d^{H,S}_{mkl}E_m + q^{E,S}_{pkl}H_p + A^{E,H}_{kl}S\\ D_n = -(\partial U_2/\partial E_n)_{\sigma,H,S} = d^{H,S}_{nij}\sigma_{ij} + \kappa^{\sigma,H,S}_{mn}E_m + m^{\sigma,S}_{pn}H_p + P^{\sigma,H}_nS\\ B_q = -(\partial U_2/\partial H_q)_{\sigma,E,S} = q^{E,S}_{qij}\sigma_{ij} + m^{\sigma,S}_{qm}E_m + \mu^{\sigma,E,S}_{pq}H_p + \theta^{\sigma,E}_qS\\ T = (\partial U_2/\partial S)_{\sigma,E,H} = -A^{E,H}_{ij}\sigma_{ij} - P^{\sigma,H}_mE_m - \theta^{\sigma,E}_pH_p + \varsigma^{\sigma,E,H}S\end{cases} \tag{5}$$