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Fiber rupture in sheared planar pantographic sheets: Numerical and experimental evidence

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ABSTRACT

Pantographic sheets are metamaterials showing some interesting mechanical features. The mechanical behavior of planar pantographic sheets has been studied by means of second gradient continuum models, see [1,2]. In [3,4] an efficient numerical code has been developed by characterizing equilibrium configurations under imposed displacement boundary conditions as those minimizing a discrete Lagrangian deformation energy. Using this model, it is possible to design experimental setups and, qualitatively and quantitatively, predict the elastic behavior of specimens built by means of 3D printing technology. In the present paper we show the first available experimental evidence obtained for sheared pantographic specimens and we show how effective and predictive is the aforementioned code. Subsequently, a simple fiber rupture mechanism is postulated and added to the initially elastic model. By adding further constitutive parameters to the previous four elastic ones, the generalized numerical model is able to predict very well the observed rupture phenomena. The promising results obtained motivate further development of proposed numerical and theoretical models and the conception of more complex experimental setups.

1. Introduction

Metamaterials are materials which are conceived by means of a theoretical effort aimed to characterize their structure by means of the equations governing their behavior. To be more precise: once a given behavior is specified by means of a set of mathematical prescriptions to be verified, the corresponding metamaterial is characterized as that material whose behavior is exactly described (under specific conditions) by the initially chosen prescriptions. The problem to be solved in this context is, in a sense, an inverse problem. Given the governing equations (or conditions) one has to find and construct the material whose behavior is predicted by them. Sometimes in literature one talk about the problem of "synthesis": given the mathematical model, is it possible to synthesize a material described by it?

Pantographic sheets are metamaterials which have been synthesized [2,1] to prove the existence of bodies behaving as second gradient materials (see [5,6]). Indeed the mechanical behavior of planar pantographic sheets exhibits the onset of internal deformation boundary layers and interact with external constraints by means of "exotic" contact interactions, see [7].

In [3,4] an efficient numerical code has been developed to get a more expedite prediction, control and design of the behavior of pantographic sheets. The concept on which this code is based considers the characterization of the kinematics of the system by means of a discrete set of Lagrangian coordinates and introduces an internal potential energy to account for stored deformation energy.

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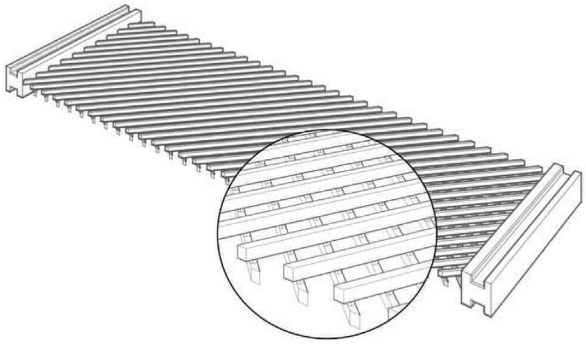

Fig. 1. Specimen constructed using 3D printing technology.

The equilibrium configurations under imposed (varying in dependence of a finite set of parameters) displacement boundary conditions are those configurations which minimize the just introduced discrete Lagrangian deformation energy.

The advantage presented by such an approach when compared with the other one based on the formulation of second gradient continuum model and its discretization with FEM can become evident in many situations. Indeed when the considered fabric does not include too many structural elements then the discretization which we propose here: (i) has a more direct physical meaning, (ii) is still able to effectively capture the global behavior of the system and (iii) is more computationally efficient.

Using the Lagrangian model presented here it has been possible to design experimental setups aimed to show the peculiar features of newly conceived metamaterial. Moreover we could qualitatively and quantitatively predict the elastic behavior of specimens built by means of 3D printing technology also in the regime of large geometrical and material nonlinearities.

In the following sections we show the first available experimental evidence obtained for sheared pantographic specimens: we designed a mechanism transforming imposed elongations into imposed shear deformations (see Fig. 1) and we characterized a large class of deformation states of pantographic sheets. Paralleling their behavior in extensional bias tests (see [2]) pantographic sheets are extremely tough also under large shear deformations. It can be shown that the code which we have developed is very effective and predictive in the elastic regime: we could get a complete correspondence between experimental data and numerical ones by simply fitting the four material parameters characterizing our model. These parameters are easily related to: (i) extensional fiber stiffness, (ii) bending fiber stiffness, (iii) shear stiffness related to fiber connectivity and (iv) material nonlinearity determining eventually non-quadratic shear deformation energy.

To be able to predict fiber rupture phenomena we subsequently postulated a simple fiber rupture mechanism. We assume that once a fiber experiences an elongation larger than a specific threshold (which is characteristic of the fiber constituting material) then it is simply broken, and the corresponding term in deformation energy is assumed to vanish. Therefore by simply adding a further constitutive parameter to the previously introduced four elastic ones we get a generalized incremental numerical model. When in the considered quasi-static imposed displacement process the rupture criterion is attained, then in the subsequent step the elastic energy is modified by imposing to zero the corresponding energy addend.

This simple model was able to predict very well the observed rupture phenomena, as it is described in the following Section 2. The results obtained and presented in Section 3 strongly motivate further development of proposed numerical and theoretical models: further rupture mechanism have been observed and need to be modeled, involving fiber interconnection rupture or partial or total loss of fiber bending stiffness, see Section 4. The consequence of the further sophistication of the used models will lead to the conception of more complex experimental setups aimed to more careful characterization of the behavior of pantographic sheets.

2. Lagrangian discrete model for pantographic structures

As discussed in full detail in [3] Lagrangian discrete models can be a viable option in the description of pantographic sheets: the possibility to predict their shapes in large deformation equilibrium configurations using said discrete models has been the object of the investigations presented in [4]. Referring the reader to just cited papers for a detailed discussion of the model and the algorithmic procedure we use to determine equilibrium configurations, we delineate here the main ideas on which we base our solution strategy when dealing with both elastic and rupture deformation phenomena.

First of all, we limit our attention to quasi-static deformation processes where the control parameters determine a well-determined class of imposed boundary displacements. The principle of virtual work will allow a facile generalization of our formulation to the fully dynamic case. Second we introduce a finite dimensional configuration space to describe the kinematics of considered sheets: it is constituted by the collection of the positions of the pivots interconnecting the two arrays of fibers, each of them being assumed extensible and flexible.

The elastic deformation energy of the sheet is assumed to be given by the sum of terms respectively (see Fig. 2) accounting for: (i) extensional energy between two adjacent pivots, (ii) bending energy of the fibers and (iii) shear energy stored in the interconnecting pivots.

We limit ourselves to consider hard-devices imposing boundary displacements to the sheets: therefore the problem of finding the equilibrium configurations in a quasi-static deformation process

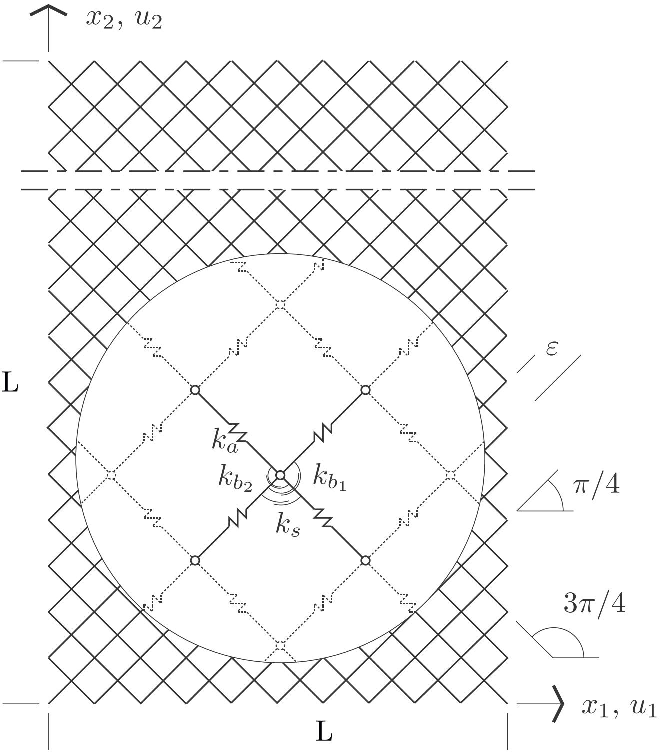

Fig. 2. Mechanical model of a pantographic 2D structure.