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Effective thermal conductivity of foam concretes: Homogenization schemes vs experimental data and FEM simulations

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ABSTRACT

This paper focuses on the prediction of thermal conductivity of foam concretes. Various analytical forms for their effective thermal conductivity according to their porosity p (air voids volume) have been first derived based on five well-known Mean-Field Homogenization (MFH) schemes. These predictions were found to be very close for low porosities but move away gradually with increasing porosity. Thus, in order to determine the best homogenization scheme predicting the effective thermal conductivity of foam concretes, MFH predictions were confronted with experimental data obtained on nineteen foam concretes and also with numerical results obtained from 3d Finite Element Method (FEM) simulations conducted on an idealized foam concrete. These comparisons have shown that the normalized effective thermal conductivity of foam concrete is closely framed by the power law $(1-p)^{3/2}$ given by the Differential scheme and by the hyperbolic law $(1-p)/(1+p/2)$ given by the Mori-Tanaka scheme.

1. Introduction

Foam concrete which is referred to also as cellular concrete is made of a cementitious matrix (a cement paste or a mortar) containing a high volume of air voids up to 0.85. This macro porosity can be obtained by adding to the cement matrix during the mixing process a preformed foam consisting of ultra-lightweight aggregates filled with air. In particular, expanded polystyrene (EPS) millimetre-size beads having a very low density (ranging between 0.015 and 0.03) are widely used today to make foam concretes. Another way to generate high porosity inside concrete consists of introducing in the cement matrix during the mixing process a gassing agent or an air entraining agent that entrains a high volume of air within concrete. This agent can be synthetic-based as aluminum powder or protein-based.

Due to their interesting properties and to their various applications, foam concretes are increasingly used in modern construction and are the focus of many researchers [1–8]. They are employed to decrease dead load of buildings when they are used in walls, partitions and screeds. They are also used as energy-absorbing materials for the protection of buried military structures and as fenders in offshore oil platforms. Besides, their high porosity makes them excellent thermal insulation materials. Within this context, the present study deals with the prediction of the thermal conductivity of foam concretes according to their microstructure.

The present paper is organized as follows. In Section 2, various analytical forms predicting the normalized effective thermal conductivity of isotropic porous materials according to their porosity are derived based on well-known Mean-Field Homogenization (MFH) schemes. In Section 3, these MFH predictions are confronted with experimental data obtained on foam concretes. In Section 4, MFH predictions are compared with numerical results obtained from Finite Element Method (FEM) simulations carried out on an idealized porous concrete. Finally, the major conclusions of the paper are summarized in Section 5.

2. MFH applied to the thermal conduction problem on isotropic porous solids

Evaluation of the effective thermal conductivity of heterogeneous materials in general and of porous solids in particular remains of interest although various approaches and models are available in literature [5,9–19]. In the present investigation, the well-known Eshelby-based Mean Field Homogenization (MFH) approach has been employed. Since foam concretes can be seen as a particular case of two-phase isotropic composites made of a solid matrix and where the inclusions phase is replaced by a random distribution of spherical air voids or EPS beads, MFH can be relevant to estimate their effective thermal conductivity according to their microstructure. Thus, we consider a Representative Volume

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Element (RVE) of an isotropic porous material made of an isotropic and homogeneous solid matrix containing spherical pores with a total volume fraction or macro porosity p. It follows that the matrix phase volume fraction is (1- p) and it is easy to check that the averages over the entire RVE (occupying a domain (ω) of a volume V), the matrix phase (ω_m) and the pores phase (ω_p) are related by:

$$\langle f\rangle_{\omega} = p\langle f\rangle_{\omega_p} + (1-p)\langle f\rangle_{\omega_m} \tag{1}$$

with $\langle f\rangle_{\omega} = \left(1/V\right)\int_{\omega} f(\underline{x})dV$ and where $\underline{x}$ is the position vector in a local frame attached to the RVE. The latter is subjected on its boundary $\left(\partial\omega\right)$ to a linear boundary temperature field $T(\underline{x})$ corresponding to a uniform temperature gradient field $\underline{E}$ $\left(T(\underline{x}) = \underline{E}\cdot\underline{x} \text{ on } \partial\omega\right)$. The micro temperature gradient field $\underline{\nabla T}(\underline{x})$ within the RVE is thus related to the macro field $\underline{E}$ through a still unknown temperature gradient concentration tensor $\underline{\underline{A}}(\underline{x})$ as follows: $\underline{\nabla T}(\underline{x}) = \underline{\underline{A}}(\underline{x}) : \underline{E}$ in ω. Moreover, it is easy to check that $\langle\underline{\nabla T}\rangle_{\omega} = \underline{E}$ and it follows that $\langle\underline{\underline{A}}\rangle_{\omega} = \underline{\underline{\delta}}$, where $\underline{\underline{\delta}}$ is the second order symmetric identity tensor.

The thermal conductivity of the isotropic solid matrix is denoted λ_m and that of pores λ_p is neglected since pores are filled with air which thermal conductivity is negligible compared to λ_m.

Assuming that the considered RVE is sufficiently wide, the Hill-Mandel's lemma stipulating the equivalence between the heterogeneous RVE and its equivalent homogenous RVE, having the same volume V and subjected on its boundary to the same linear temperature field $T(\underline{x}) = \underline{E}\cdot\underline{x}$ as the heterogeneous RVE but characterized by a still unknown effective thermal conductivity denoted λ_{hom}, allows to write that:

$$\langle\underline{\Delta T\cdot q}\rangle_{\omega} = \langle\underline{\Delta T}\rangle_{\omega}\cdot\langle\underline{q}\rangle_{\omega} = \underline{E}\cdot\underline{Q} \tag{2}$$

where $\underline{q}$ and $\underline{Q}$ are respectively the micro heat flow vector of the heterogeneous RVE and the macro heat flow vector of its equivalent homogenous RVE. The latter is therefore given by:

$$\underline{Q} = \langle\underline{q}\rangle_{\omega} = \langle-\underline{\lambda}\cdot\underline{\Delta T}\rangle_{\omega} = \langle-\underline{\lambda}\cdot\underline{\underline{A}}\rangle_{\omega}\cdot\underline{E} \tag{3}$$

Then, since $\underline{Q} = -\underline{\lambda}_{\text{hom}}\cdot\underline{E}$, it follows that the effective thermal conductivity λ_{hom} is given by:

$$\lambda_{hom} = \langle\underline{\lambda}\cdot\underline{\underline{A}}\rangle_{\omega} \tag{4}$$

In the particular case of isotropic porous solids,$\lambda_p = 0$ and the previous formula is reduced to:

$$\lambda_{hom} = \lambda_m\left(\underline{\underline{\delta}} - p\langle\underline{\underline{A}}\rangle_{\omega_p}\right) \tag{5}$$

Thus, to determine the effective thermal conductivity λ_{hom} of an isotropic porous solid, we need to compute or to estimate the average of the temperature gradient concentration tensor over the pores phase $\langle\underline{\underline{A}}\rangle_{\omega_p}$. For the crude approximation: $\langle\underline{\underline{A}}\rangle_{\omega_p} = \underline{\underline{\delta}}$ which means that the temperature field is homogeneous inside the heterogeneous RVE (*i.e.* $\underline{\nabla T}(\underline{X}) = \underline{E}$ in ω), we obtain:

$$\lambda_{hom}{}^{Voigt}(p) = \lambda_m(1-p) \tag{6}$$

This homogenization scheme is commonly referred to as the Voigt model and it provides the upper bound for the effective thermal conductivity of isotropic porous solids according to their porosity.

Mean-Field Homogenization is classically based on Eshelby's tensors [20]. The latter are derived from the resolution of auxiliary thermo-elastic problems and are used to estimate the average elastic strain and stress, temperature gradient and heat flow fields inside inclusions (pores here). Eshelby's concentration tensors account also for inclusions shape and orientation. In the case of isotropic two-phase composites and particularly isotropic porous solids, explicit formulae can be derived for Eshelby's tensors leading to explicit estimates for the effective elastic moduli and thermal conductivity of these materials.

2.1. Eshelby's and Hill's tensors of the auxiliary thermal conduction problem

For a single ellipsoidal heterogeneity having an isotropic thermal conductivity λ_I and occupying a domain (I) embedded in an infinite homogeneous matrix having an isotropic thermal conductivity λ_m and subjected to a uniform temperature gradient field $\underline{E}$ on its boundary (i.e.$\underline{\nabla T}(\underline{x}) = \underline{E}.\underline{x}at\infty$), Eshelby [20] showed that the temperature gradient field inside (I) is uniform and that: $\underline{\nabla T}(\underline{x}) = \underline{\underline{H}}^I : \underline{E}$ inside (I), where $\underline{\underline{H}}^I$ is the Eshelby's temperature gradient concentration tensor related to this auxiliary thermal conduction problem and which is given by the following expression: $\underline{\underline{H}}^I = \left[\underline{\underline{\delta}} + (\lambda_I - \lambda_m)\underline{\underline{P}}^I\right]^{-1}$,with $\underline{\underline{P}}^I$ is the Hill's polarization tensor [21] associated to the Eshelby's tensor $\underline{\underline{H}}^I$. In the particular case of a single spherical pore $\left(\lambda_I = \lambda_p = 0\right)$ embedded in an infinite isotropic matrix, the Hill's tensor is given by: $\underline{\underline{P}}^I = \frac{1}{3\lambda_m}\underline{\underline{\delta}}$ and the Eshelby's temperature gradient concentration tensor is reduced to: $\underline{\underline{H}}^I = \frac{3}{2}\underline{\underline{\delta}}$.

Hereafter, analytical formulae for the effective thermal conductivity of isotropic porous solids are derived based on four well-known Mean-Field Eshelby-based Homogenization schemes: the Dilute method, the Mori–Tanaka model, the Self Consistent model and the Differential method.

2.2. The Dilute scheme

The Dilute scheme or method does not account for interaction between pores. It stipulates that each pore (I) behaves like an isolated pore in an infinite matrix subjected to a uniform temperature gradient field $\underline{E}$ on its boundary, which corresponds to the Eshelby's single heterogeneity problem.

Thus, $\langle\underline{\underline{A}}\rangle_{\omega_p}{}^{Dilute} = \underline{\underline{H}}^I = \frac{3}{2}\underline{\underline{\delta}}$ in the case of isotropic porous solids and it follows that:

$$\lambda_{hom}{}^{Dilute}(p) = \lambda_m(1 - \frac{3}{2}p) \tag{7}$$

2.3. The Mori-Tanaka scheme

According to this model [22,23], each pore (I) behaves like an isolated pore in an infinite matrix subjected on its boundary to the average of the temperature gradient field over the matrix phase $\langle\underline{\nabla T}\rangle_{\omega_m}$. Consequently, the gradient temperature field inside (I) is uniform and is given by: $\underline{\nabla T}(\underline{x}) = \underline{\underline{H}}^I : \langle\underline{\nabla T}\rangle_{\omega_m}$ inside (I).

Thus, $\langle\underline{\nabla T}\rangle_{\omega_p} = \frac{3}{2(1+\frac{p}{2})} : \underline{E}$ and it follows that:

$$\lambda_{hom}{}^{M-T}(p) = \lambda_m\frac{1-p}{1+\frac{p}{2}} \tag{8}$$

We note also that this hyperbolic form according to porosity given by the Mori-Tanaka scheme corresponds in the case of isotropic porous solids to the Hashin-Shtrikman upper bound [24] and also to the estimate of the Hashin Composite Sphere model [25]. It coincides also in this case with the Maxwell's model [9].

2.4. The Self-Consistent scheme

The Self-Consistent scheme [21,26] can be defined directly by assuming that each pore (I) is embedded in a fictitious homogeneous and infinite matrix possessing the effective unknown thermal conductivity λ^{SC} and subjected to a uniform temperature