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On the propagation waves in the theory of thermoelasticity with microtemperatures

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ABSTRACT

The present paper studies the propagation of plane time harmonic waves in an infinite space filled by a thermoelastic material with microtemperatures. It is found that there are seven basic waves traveling with distinct speeds: (a) two transverse elastic waves uncoupled, undamped in time and traveling independently with the speed that is unaffected by the thermal effects; (b) two transverse thermal standing waves decaying exponentially to zero when time tends to infinity and they are unaffected by the elastic deformations; (c) three dilatational waves that are coupled due to the presence of thermal properties of the material. The set of dilatational waves consists of a quasi-elastic longitudinal wave and two quasi-thermal standing waves. The two transverse elastic waves are not subjected to the dispersion, while the other two transverse thermal standing waves and the dilatational waves present the dispersive character. Explicit expressions for all these seven waves are presented. The Rayleigh surface wave propagation problem is addressed and the secular equation is obtained in an explicit form. Numerical computations are performed for a specific model, and the results obtained are depicted graphically.

1. Introduction

Ieşan and Quintanilla [1] have developed a linear theory of thermoelasticity with microtemperatures by taking into consideration of the microstructure of the body and assuming that each microelement possesses a microtemperature. Such a theory deserves to study the effects of the temperature wave propagation in a nanomaterial which allows for variation of thermal properties at a microstructure level. The microstructural theories are now used into the experimental research for determining the effective thermal conductivity properties of nanomaterials (see, for example, Straughan [2] and the references therein).

Grot [3] was the first to take into consideration the inner structure of a body in order to develop a thermodynamic theory for thermoelastic materials where microelements, in addition to classic microdeformations, possess microtemperatures. Riha [4] further developed a study concerning heat conduction in thermoelastic materials with inner structure.

The theory of thermoelasticity with microtemperatures has attracted much attention in connection with the study of the basic qualitative properties of solutions to the problems relating to various thermomechanical situations (see, for example, [5–14]). Recently, Ciarletta et al. [15] investigate a model for a rigid heat conductor which allows for variation of thermal properties at a microstructural level and they examine how the solution depends on changes in coupling coefficients between the macro and microthermal level.

Vadasz et al. [16] suggest that the thermal wave effects should be taken into account within the question of interpreting experimental results regarding the measurements on the thermal conductivity in nanomaterials.

In this last connection, in the present paper we consider the linear theory of thermoelasticity with microtemperatures as developed in [1] and address the wave propagation problem in the class of solutions with finite energy. Namely, we are considering here damped and undamped in time wave solutions. We prove that there are seven basic waves traveling with distinct speeds: (a) two transverse elastic waves uncoupled, undamped in time and traveling independently with a speed that is unaffected by the thermal effects; (b) two transverse thermal standing waves decaying exponentially to zero when time tends to infinity and they are unaffected by the elastic deformations; (c)

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three dilatational waves that are coupled due to the presence of thermal properties of the material. Within the framework of the dilatational set there is a quasi-elastic longitudinal wave and two quasi-thermal standing waves. The two transverse elastic waves are not subjected to the dispersion, while the other two transverse thermal standing waves and the dilatational waves present the dispersive character. Explicit expressions for all these seven waves are presented. The propagation of the Rayleigh surface waves is studied in a half space filled by a thermoelastic material with microtemperatures and the secular equation is obtained in an explicit expression. Numerical computations are performed for a specific model, and the results obtained are depicted graphically.

Recently, Steeb et al. [17] studied the propagation of waves of assigned frequency in an infinite thermoelastic medium with microtemperatures, while Kumar et al. [18] considered the problem of reflection and transmission of waves at an interface of elastic and microstretch thermoelastic solids with microtemperatures. The authors consider waves with real-valued frequency and complex-valued wave number, being led to solutions with unbounded energy.

2. Basic equations

Throughout this section B is a bounded regular region of three-dimensional Euclidean space. We let ∂B denote the boundary of B, and designate by $\mathbf{n}$ the outward unit normal on ∂B. We assume that the body occupying B is a linearly elastic material which possesses thermal variation at a microstructure level. The body is referred to a fixed system of rectangular Cartesian axes Ox_i (i = 1, 2, 3). Throughout this paper Latin indices have the range 1, 2, 3, Greek indices have the range 1, 2 and the usual summation convention is employed. We use a subscript preceded by a comma for partial differentiation with respect to the corresponding coordinate and a superposed dot denotes partial differentiation with respect to time.

The temperature at a point $\mathbf{x}$ of the body depends on a temperature $\theta(\mathbf{x}, t)$, which may be thought of as an averaged temperature at $\mathbf{x}$, and three microtemperatures $w_i(\mathbf{x}, t)$ which contribute to the thermal microstructure of the material. The deformation of a body can be described by means of three, namely, the displacement vector field $\mathbf{u}$, the microtemperature vector field $\mathbf{w}$ and the temperature variation T, measured from the constant absolute temperature T_0 (>0), over $B \times (0, \infty)$.

Within the framework of the linear theory developed by Ieşan and Quintanilla [1], the constitutive equations for a homogeneous and isotropic thermoelastic solid with microtemperatures are

$$\begin{aligned} & t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} - \beta T\delta_{ij}, \\ & \varrho\eta = \beta e_{rr} + aT, \\ & \varrho\varepsilon_i = -bw_i, \\ & q_i = kT_{,i} + \kappa_1 w_i \\ & Q_i = (\kappa_1 - \kappa_2)w_i + (k - \kappa_3)T_{,i}, \\ & q_{ij} = -\kappa_4 w_{r,r}\delta_{ij} - \kappa_5 w_{i,j} - \kappa_6 w_{j,i}, \end{aligned} \tag{1}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{2}$$

Here, t_{ij} are the components of the stress tensor, ϱ is the reference mass density, η is the entropy per unit mass, ε_i are the components of the first moment of energy vector, q_i are the components of the heat flux vector, Q_i are the components of the mean heat flux vector, q_{ij} are the components of the first heat flux moment vector, e_{ij} are the components of the strain tensor, u_i are the components of the displacement vector, w_i are the components of the microtemperature vector, T is the temperature variation, λ, μ, β, a, b, k and κ_r (r = 1, 2, ..., 6) are constant constitutive coefficients and δ_{ij} is the Kronecker delta.

The fundamental system of field equations of the linear theory of thermoelasticity with microtemperatures consists of [1]:

– the equations of motion

$$t_{ji,j} + \varrho f_i = \varrho\ddot{u}_i, \tag{3}$$

– the balance energy

$$\varrho T_0\dot{\eta} = q_{i,i} + \rho S, \tag{4}$$

– the first moment of energy

$$\varrho\dot{\varepsilon}_i = q_{ji,j} + q_i - Q_i + \varrho M_i, \tag{5}$$

where f_i are the components of the body force vector, M_i are the components of the first heat source moment vector and S is the heat supply.

The components of surface traction t_i, the heat flux q and the components of the first heat flux moment Λ_i at a regular point $\mathbf{x}$ of the boundary ∂B are given by

$$t_i = t_{ji}n_j, q = q_i n_i, \quad \Lambda_i = q_{ji}n_j, \tag{6}$$

where $n_j = \cos(\mathbf{n_x}, Ox_j)$ and $\mathbf{n_x}$ is the unit vector of the outward normal to ∂B at $\mathbf{x}$.

Within the context of linear theory of thermoelasticity considered in [1], the Clausius–Duhem inequality reduces to

$$q_i T_{,i} - T_0 q_{ji} w_{i,j} - T_0(Q_i - q_i)w_i \geq 0, \tag{7}$$