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Time-dependent interfacial sliding in nano-fibrous composites under longitudinal shear

Xu Wang [a], Cuiying Wang [a], Peter Schiavone [b,*]

[a] School of Mechanical and Power Engineering, East China University of Science and Technology, 130 Meilong Road, Shanghai 200237, China
[b] Department of Mechanical Engineering, University of Alberta, 10-203 Donadeo Innovation Centre for Engineering, 9211-116 Street NW, Edmonton, Alberta T6G 1H9, Canada

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ABSTRACT

We use complex variable methods to analyze time-dependent deformations of an isolated elastic nano-fiber embedded in an infinite elastic matrix under longitudinal shear subjected to uniform stress at infinity. In order to incorporate nanoscale size-effects into our continuum-based model, the nano-fiber and the matrix are each endowed with separate and distinct surface elasticities described by the Gurtin–Murdoch model. The fiber/matrix interface is allowed to slide via a diffusion-controlled mechanism. We show that the characteristic relaxation time and time-dependent stress distribution in the composite can be described completely by three size-dependent parameters and a size-independent mismatch parameter. Further, we note that the internal stress field is spatially uniform yet size- and time-dependent. Finally, we obtain the effective anelastic and size-dependent shear modulus of the fibrous composite using the mean-field method of Mori–Tanaka.

1. Introduction

Phenomena associated with time-dependent interfacial sliding in polycrystals and composite materials have been studied extensively (see, for example, Refs. [6–9,15,18]). Time-dependent sliding can be attributed to either an artificially introduced viscous interface layer designed to tailor damping performance of the composite or to local diffusion on a length scale comparable to the size of the asperities present in the interface [12]. In this paper, we adopt the idea of a diffusion-controlled mechanism in which the gradient of the normal stress along the interface results in the diffusion of atoms from at least one of the adjoining materials to the interface thus giving rise to interfacial sliding.

It is well-known that surface/interface stresses, tensions and energies become significant when the dimensions of the fibers lie in the nanometer-range [14]. To accommodate surface (and hence nanoscale) effects into our model of deformation we appeal to the theory of surface elasticity. The most celebrated continuum-based surface elasticity model was first proposed by Gurtin et al. [3–5] and recently clarified by Ru [13]. In the Gurtin–Murdoch model, the surface is regarded as a thin elastic membrane perfectly bonded to the bulk [2,10,16].

In this paper, we first investigate the deformation of a single elastic nano-fiber embedded in an infinite elastic matrix subjected to remote uniform anti-plane shear stresses. The surface of the nano-fiber and that of the matrix are endowed with separate and distinct Gurtin–Murdoch surface elasticities and the fiber/matrix interface is allowed to slide via the diffusion-controlled mechanism mentioned above as identified by Raj and Ashby [12]. As in He and Lim [7,8], our analysis is confined to a quasi-static process. Using complex variable methods, we develop the time-dependent displacement and stress distributions in the composite. We show that the characteristic relaxation time and the aforementioned elastic distributions are completely determined by three size-dependent parameters and one size-independent mismatch parameter. The normalized relaxation time remains size-dependent due to the incorporation of surface elasticities. The effective time-dependent and size-dependent anti-plane shear modulus of the composite with finite nano-fiber concentration is subsequently obtained using the Mori–Tanaka mean-field method [1,11,19].

2. Basic formulation

2.1. The bulk elasticity

In what follows, unless otherwise stated, Latin indices i, j, k take the values 1–3 and we sum over repeated indices. In a Cartesian coordinate system $\{x_i\}$, the equilibrium equation and the

* Corresponding author.
E-mail addresses: xuwang@ecust.edu.cn (X. Wang), cuiyingwang@126.com (C. Wang), p.schiavone@ualberta.ca (P. Schiavone).

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stress–strain law in a linearly elastic, homogeneous and isotropic bulk solid are given by

$$\sigma_{ij,j}=0,\quad \sigma_{ij}=2\mu\varepsilon_{ij}+\lambda\varepsilon_{kk}\delta_{ij},\\ \varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}),\tag{1}$$

where λ and μ are the Lame constants, σ_{ij} and ε_{ij} are, respectively, the Cartesian components of the stress and strain tensors in the bulk material, μ_i is the i-th component of the displacement vector and δ_{ij} is the Kronecker delta.

In the case of anti-plane shear deformations of an isotropic elastic material, the two shear stress components σ_{31} and σ_{32}, the out-of-plane displacement $w=u_3(x_1,x_2)$ and the stress function ϕ can be expressed in terms of a single analytic function $f(z)$ of the complex variable $z=x_1+\mathrm{i}x_2$ as

$$\sigma_{32}+\mathrm{i}\sigma_{31}=\mu f'(z),\quad \phi+\mathrm{i}\mu w=\mu f(z).\tag{2}$$

Let t_3 be the only non-zero traction component along the x_3-direction on a given boundary curve L. It can be shown that if s is the arc-length measured along L so that the enclosed material remains on the left with increasing s then [17],

$$t_3=-\frac{\mathrm{d}\phi}{\mathrm{d}s}.\tag{3}$$

2.2. The surface elasticity

The equilibrium conditions on a surface incorporating interface/surface elasticity can be expressed as [3–5,13].

$$[\sigma_{\alpha j}n_j\underline{e}_\alpha]+\sigma^s_{\alpha\beta,\beta}\underline{e}_\alpha=0,\quad \text{(tangential direction)}\\ [\sigma_{ij}n_in_j]=\sigma^s_{\alpha\beta}\kappa_{\alpha\beta},\quad \text{(normal direction)}\tag{4}$$

where n_i represent the Cartesian components of the unit normal vector to the surface, [*] denotes the jump of the respective quantity across the surface and $\sigma^s_{\alpha\beta}$ and $\kappa_{\alpha\beta}$ are, respectively, the components of the surface stress tensor and the curvature tensor of the surface. In addition, the constitutive equations on the isotropic surface are given by

$$\sigma^s_{\alpha\beta}=\sigma_0\delta_{\alpha\beta}+2(\mu_s-\sigma_0)\varepsilon^s_{\alpha\beta}+(\lambda_s+\sigma_0)\varepsilon^s_{\gamma\gamma}\delta_{\alpha\beta},\tag{5}$$

where $\varepsilon^s_{\alpha\beta}$ are the components of the surface strain tensor, σ_0 is the surface tension and λ_s and μ_s are the two surface Lame constants.

We mention that in Eqs. (4) and (5), the Greek indices α, β and γ take on values of the surface components. For example, in the case of circular cylindrical fibers, α, β, γ each take on the values θ, z.

2.3. Time-dependent interfacial sliding

In the presence of a tangential traction of magnitude τ, diffusion-controlled time-dependent sliding will take place along the fiber/matrix interface according to the following linear law [12]

$$\tau=\eta\dot{\delta}.\tag{6}$$

Here, δ is the displacement jump across the interface, the overdot denotes differentiation with respect to time t and η is the interface slip coefficient which can be measured empirically.

3. Interfacial sliding of an isolated nano-fiber

Consider first the case of an isolated nano-fiber embedded in an infinite matrix. The linearly elastic materials occupying the fiber and the matrix are assumed to be homogeneous and isotropic with associated shear moduli μ_1 and μ_2, respectively. We represent the matrix by the domain S_2 and assume that the fiber occupies a circular region S_1 of radius R with center at the origin. The fiber/matrix interface is denoted by the curve L. In what follows, the subscripts 1 and 2 (or the superscripts (1) and (2)) are used to identify the respective quantities in S_1 and S_2. The matrix is subjected to remote uniform anti-plane shear stresses $(\sigma^\infty_{31},\sigma^\infty_{32})$. Separate surface elasticities are simultaneously incorporated in the descriptions of the surface of the fiber and that of the surface of the adjoining matrix. In addition, the fiber/matrix interface is allowed to slide via the diffusion-controlled mechanism described above. The stresses in the fiber and the matrix change gradually during interfacial sliding under quasi-static conditions. Consequently, Eq. (1) remains valid for each of the two constituent phases.

We further assume that the interface L is coherent with respect to either the inhomogeneity or the matrix. It then follows from Eqs. (4) and (5) that the boundary conditions on the surface of the fiber and matrix can be written, respectively, as

$$\mu_1\frac{\mathrm{d}w_1}{\mathrm{d}n}-\sigma^-_{3n}=(\mu^{(1)}_s-\sigma^{(1)}_0)\frac{\mathrm{d}^2w_1}{\mathrm{d}s^2},\tag{7a}$$

on the surface of the fiber,

$$\sigma^+_{3n}-\mu_2\frac{\mathrm{d}w_2}{\mathrm{d}n}=(\mu^{(2)}_s-\sigma^{(2)}_0)\frac{\mathrm{d}^2w_2}{\mathrm{d}s^2},\tag{7b}$$

on the surface of the matrix.

Here, n denotes the outward unit normal to L, s is the arc parameter measured counterclockwise from n, and

$$\sigma^-_{3n}=\sigma^+_{3n}=\eta(\dot{w}_2-\dot{w}_1).\tag{8}$$

In view of Eq. (3), the above interface conditions along $|z|=R$ can be expressed in terms of the tangential derivative of the displacement and stress function along the interface as follows

$$\eta(\dot{w}_1-\dot{w}_2)-\frac{\mathrm{d}\phi_1}{\mathrm{d}s}=(\mu^{(1)}_s-\sigma^{(1)}_0)\frac{\mathrm{d}^2w_1}{\mathrm{d}s^2},\\ \frac{\mathrm{d}\phi_2}{\mathrm{d}s}-\eta(\dot{w}_1-\dot{w}_2)=(\mu^{(2)}_s-\sigma^{(2)}_0)\frac{\mathrm{d}^2w_2}{\mathrm{d}s^2},\\ |z|=R.\tag{9}$$

Eq. (9) can be expressed in terms of $f_1(z)$, $f_2(z)$ and their analytic continuations $\bar{f}_1(R^2/z)$, $\bar{f}_2(R^2/z)$ in each phase as:

$$\begin{aligned}&\eta\left[\dot{f}^+_1(z)+\dot{\bar{f}}^+_2(R^2/z)\right]+\frac{\mu_1z}{R}f'^+_1(z)\\&+(\mu^{(1)}_s-\sigma^{(1)}_0)\left[\frac{z}{R^2}f'^+_1(z)+\frac{z^2}{R^2}f''^+_1(z)\right]\\&=\eta\left[\dot{\bar{f}}^-_1(R^2/z)+\dot{f}^-_2(z)\right]+\frac{\mu_1R}{z}\bar{f}'^-_1(R^2/z)\\&+(\mu^{(1)}_s-\sigma^{(1)}_0)\left[\frac{1}{z}\bar{f}'^-_1(R^2/z)+\frac{R^2}{z^2}\bar{f}''^-_1(R^2/z)\right],\\&\eta\left[\dot{f}^+_1(z)+\dot{\bar{f}}^+_2(R^2/z)\right]-\frac{\mu_2R}{z}\bar{f}'^+_2(R^2/z)\\&+(\mu^{(2)}_s-\sigma^{(2)}_0)\left[\frac{1}{z}\bar{f}'^+_2(R^2/z)+\frac{R^2}{z^2}\bar{f}''^+_2(R^2/z)\right]\\&=\eta\left[\dot{\bar{f}}^-_1(R^2/z)+\dot{f}^-_2(z)\right]-\frac{\mu_2z}{R}f'^-_2(z)\\&+(\mu^{(2)}_s-\sigma^{(2)}_0)\left[\frac{z}{R^2}f'^-_2(z)+\frac{z^2}{R^2}f''^-_2(z)\right],\quad |z|=R.\end{aligned}\tag{10}$$