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On periodic indentation of a rigid solid occupying a wavy surface moving on multiferroic materials

Yue-Ting Zhou, Zheng Zhong*

School of Aerospace Engineering and Applied Mechanics, Tongji University, Shanghai 200092, PR China

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ABSTRACT

The present article is concerned with a 2D dynamic contact problem of a rigid solid occupying a wavy surface moving on anisotropic multiferroic materials. Three bulk wave velocities for anisotropic multiferroic materials are obtained. Five harmonic functions are suggested to derive the general solutions of anisotropic multiferroic governing equations based on the generalized Almansi's theorem. The contact length and various stresses, electric displacements, and magnetic inductions can be given in the whole half-plane analytically. Figures are plotted to gain an insight of how the velocity and the elastic coefficient influence the contact performance of anisotropic multiferroic materials. Numerical results show that the contact length can be enlarged by escalating the velocity of the rigid solid.

1. Introduction

It is found that a wide class of crystals and emerging composite materials will possess simultaneously piezoelectric, piezomagnetic and magneto-electric effects, which are thereby classified as multiferroic solids. These multiferroic materials have the ability of energy conversion among electric, magnetic and mechanical fields [15], which can be used as magnetic field probes, electric packing, acoustic, hydrophones, medical, ultrasonic image processing, sensors and actuators [24].

Heretofore, mechanical behaviors especially contact behaviors of multiferroic materials gained intensive academic efforts since solutions of contact problems lay the foundation of the indentation techniques, which are widely used to measure various properties of newly designed materials [13,5]. For example, Rogowski and Kalinski [25] examined a truncated conical indenter acting on a multiferroic half space. Elloumi et al. [7,8] concerned the frictional sliding contact problem between a homogeneous magneto-electro-elastic half-space and a perfectly conducting rigid flat or circular punch under the action of magneto-electro-mechanical loads with an analytical closed-form solution obtained for the normal contact stresses, electric displacement and magnetic induction distributions. Recently, Li et al. [18,17] presented the fundamental magneto-electro-elastic field in a multiferroic half-space indented by a half-infinite punch or a rigid flat-ended elliptic punch with the aid of the recent progress in the method of potential theory [9,10]. The multiferroic materials involved in these studies are transversely isotropic. Zhou and Zhong [28] derived the solutions of the frictional sliding contact between a rigid punch and anisotropic multiferroic materials with the effect of the volume fraction of piezoelectric phase on contact performance revealed.

The contact problems mentioned above were all in steady state. In the practical engineering, there is sliding between the contacting bodies. Thus, dynamic contact is an important category of contact problem. In particular, Afferrante and Ciavarella [1] pointed out that frictional motion may generate non-uniformities, noise and vibrations of interest in science and technology, ranging from nanotribology [23] to "squeal" or "hot-spotting" in brakes or clutches [14,19]. For a given coefficient, there exists a critical speed above which the frictionally generated heat makes the sliding systems, such as brakes, unstable [2]. As promising alternative materials, functionally graded materials (FGM) [11] possessing continuously graded properties have been proposed to relieve instability of the dynamic contact [12]. Lee and Jang [16] performed an analytical analysis of dynamic contact problem of the FGM half plane sliding against a rigid non-conducting or a homogeneous conducting body at an arbitrary speed. They revealed that the non-homogeneous parameters of the elastic modulus and thermal expansion coefficient greatly affect the variation of the critical speed and can be selected to adjust it. The materials in these dynamic contact problems were purely elastic. There were a few investigations on dynamic contact problems of multiferroic materials. Zhou and Lee

* Corresponding author.
E-mail address: zhongk@tongji.edu.cn (Z. Zhong).

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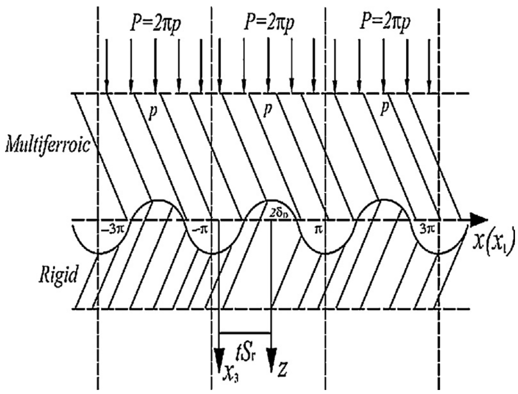

Fig. 1. Partial contact between a moving rigid solid with a wavy surface and multiferrioc materials.

[26,27] investigated dynamic contact between a multiferroic plane and a moving indenter with a flat or cylindrical profile, and found that the speed as well as the magneto-electro properties of the indenter is a strong influential factor to the contact performance, in which the dynamic contact is of a single zone case. The general case of multiple zone contact involving multiferroic materials with moving velocity concerned needs to be studied. In addition, anisotropy of multiferroic materials [22] should also be considered.

Inspired by the above-mentioned reasons, a dynamic contact analysis of anisotropic multiferroic materials subject to a moving rigid solid is conducted with three bulk wave velocities presented. The anisotropic multiferroic governing equations are solved based on the potential theory. A parametric study is conducted in a systematic manner to show how the velocity and the elastic coefficient ratio affect the distributions of various physical quantities. Figures show that the velocity and the elastic coefficient are key factors to influence the contact length between anisotropic multiferroic materials and the rigid solid; and the effects of the two factors mentioned above on the magnetic induction distribution are similar to those on the electrical displacement distribution.

2. Statement of problem

Anisotropic multiferroic materials originally placed in the Cartesian coordinates $x_i (i = 1–3)$ are in partial contact with a rigid solid with a wavy, hard surface (Fig. 1) with the amplitude of the profile being small compared with the wave length. The rigid solid moves at a constant speed S_r smoothly. In the case when no free electric charge, electric current and body force exist, the basic equations for anisotropic multiferroic materials can be written as

$$\sigma_{mN,N} = \begin{cases} \rho \dfrac{\partial^2 u_m}{\partial t^2}, & N = 1\text{–}3 \\ 0 & N = 4, 5 \end{cases}, \tag{1}$$

where ρ stands for the mass density, t represents the time variable, the comma denotes partial differentiation, and the constitutive equations are given by

$$\sigma_{mN} = C_{mNIj} u_{I,j}, \tag{2}$$

where the extended stresses, extended displacements and extended elastic coefficients take the following forms [21]:

$$\sigma_{mN} = \begin{cases} \sigma_{mn}, & N \le 3 \\ D_m & N = 4 \\ B_m & N = 5 \end{cases}, \tag{3}$$

$$u_I = \begin{cases} u_i, & I \le 3 \\ \phi, & I = 4 \\ \psi, & I = 5 \end{cases}, \tag{4}$$

$$C_{mNIj} = \begin{cases} C_{mnij}, & N, I \le 3 \\ e_{jmn}, & N \le 3, I = 4 \\ h_{jmn}, & N \le 3, I = 5 \\ e_{mij}, & N = 4, I \le 3 \\ -\varepsilon_{mj}, & N = 4, I = 4 \\ -d_{mj}, & N = 4, I = 5 \\ h_{mij}, & N = 5, I \le 3 \\ -d_{mj}, & N = 5, I = 4 \\ -\mu_{mj}, & N = 5, I = 5 \end{cases}, \tag{5}$$

where u_i, ϕ and ψ are the mechanical displacements, electric potential and magnetic potential; σ_{mn}, D_m and B_m are the stress components, electric displacements and magnetic inductions; C_{mnij}, e_{jmn} and ε_{mj} are the elastic coefficients, piezoelectric coefficients and dielectric coefficients; and h_{jmn}, d_{mj} and μ_{mj} are the piezomagnetic coefficients, magnetoelectric coefficients and magnetic permeability coefficients.

It should also be pointed out that the following symmetry relations hold true:

$$\begin{aligned} &C_{mnij} = C_{nmij} = C_{mnji} = C_{ijmn}, \qquad e_{jmn} = e_{jnm}, \quad h_{jmn} = h_{jnm}, \\ &\varepsilon_{mj} = \varepsilon_{jm}, d_{mj} = d_{jm}, \mu_{mj} = \mu_{jm}. \end{aligned} \tag{6}$$

3. General solution of anisotropic multiferroic materials

One may introduce the following Galilean transformation:

$$x = x_1 \pm S_r t, \quad y = x_2, \quad z = x_3, \tag{7}$$

with "+" representing that the rigid indenter moves to the left, while "−" to the right.

For a plane-strain problem with all physical quantities being independent on y, substituting constitutive relationships Eq. (2) into Eq. (1) and considering Eq. (7), one gets the governing equations for anisotropic multiferroic materials as follows:

$$\mathbf{H}_{5\times5} \times \{ u \quad v \quad w \quad \phi \quad \psi \}^T = \mathbf{0}, \tag{8}$$

with superscript T representing the transpose, the mechanical displacements u_i replaced as u, v and w, and $\mathbf{H}_{5\times5}$ given as

$$\left\{ \begin{array}{ccc} \alpha_1 \dfrac{\partial^2}{\partial x^2} + C_{55} \dfrac{\partial^2}{\partial z^2} & C_{16} \dfrac{\partial^2}{\partial x^2} + C_{45} \dfrac{\partial^2}{\partial z^2} & \alpha_2 \dfrac{\partial^2}{\partial x \partial z} \\ C_{16} \dfrac{\partial^2}{\partial x^2} + C_{45} \dfrac{\partial^2}{\partial z^2} & \alpha_5 \dfrac{\partial^2}{\partial x^2} + C_{44} \dfrac{\partial^2}{\partial z^2} & \alpha_6 \dfrac{\partial^2}{\partial x \partial z} \\ \alpha_2 \dfrac{\partial^2}{\partial x \partial z} & \alpha_6 \dfrac{\partial^2}{\partial x \partial z} & \alpha_9 \dfrac{\partial^2}{\partial x^2} + C_{33} \dfrac{\partial^2}{\partial z^2} \\ \alpha_3 \dfrac{\partial^2}{\partial x \partial z} & \alpha_7 \dfrac{\partial^2}{\partial x \partial z} & e_{15} \dfrac{\partial^2}{\partial x^2} + e_{33} \dfrac{\partial^2}{\partial z^2} \\ \alpha_4 \dfrac{\partial^2}{\partial x \partial z} & \alpha_8 \dfrac{\partial^2}{\partial x \partial z} & h_{15} \dfrac{\partial^2}{\partial x^2} + h_{33} \dfrac{\partial^2}{\partial z^2} \end{array} \right.$$