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Basic solution of a plane rectangular crack in a 3-D infinite orthotropic elastic material

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ABSTRACT

The solution of a plane rectangular crack in a 3-D infinite orthotropic elastic material is investigated by means of the generalized Almansi's theorem and the Schmidt method in the present paper. By using the 2-D Fourier transform and defining the jumps of displacement components across the crack surfaces as the unknown variables, three pairs of dual integral equations are derived. To solve the dual integral equations, the jumps of the displacement components across the crack surface are directly expanded in a series of Jacobi polynomials. Numerical examples are provided to show the effects of the geometric shape of the rectangular crack on the stress intensity factors in an orthotropic elastic material.

1. Introduction

Cracked structural members are occasionally subjected to stress loading. In an engineering viewpoint, obtaining the associated stress intensity factors is of great interest. Recently, three-dimensional fracture problems in composite materials have received much attention (Itou, 1994, 1999, 2002, 2007; Liu et al., 2001; Wang and Shen, 2002; Chen et al., 2004; Zhao et al., 2007; Zhu and Qin (2007); Liu et al., 2012; Zhou et al., 2012).

In the above literatures, the exact solutions were obtained and the crack-shaped was often assumed to be circular cavity (Itou, 1994), square-shaped crack (Pan and Yuan, 2000), rectangular crack (Itou, 1999, 2002; Liu et al., 2012; Zhou et al., 2012), penny-shaped crack (Pan and Yuan, 2000; Chen et al., 2004), elliptical crack (Liu et al., 2001) or cylindrical crack (Itou, 2007). Compared to the theoretical works, only a few open literatures studied an arbitrary-shaped three-dimensional problem. For instances, Pan and Yuan (2000) presented a boundary element analysis of linear elastic fracture mechanics in three-dimensional cracks of anisotropic solids. Zhao et al. (2007) discussed an arbitrary-shaped three-dimensional plane crack embedded in an infinite transversely isotropic magnetoelectroelastic medium by using Green's functions and boundary integral equation approach, and some useful conclusions were obtained. The problems of rectangular crack in isotropic elastic materials by using the Schmidt method have been solved by Itou (1999, 2001, 2002). Zhou et al. (2012) analyzed a 3-D rectangular limited-permeable crack or two 3-D rectangular limited-permeable cracks in transversely isotropic piezoelectric materials by using Schmidt method and generalized Almansi's theorem. Liu et al. (2012) dealt with four 3-D rectangular limited-permeable cracks in transversely isotropic piezoelectric materials by using Schmidt method and generalized Almansi's theorem.

In all these studies, the material properties are assumed to be isotropic or transversely isotropic materials. However, because of the nature of the techniques used in processing, the material properties are seldom orthotropic. Recently, composite materials, which are essentially orthotropic materials, have attracted attention due to their high strength and relative lightness. Therefore, the orthotropic properties should be considered in studying for the fracture mechanics. For example, Ma et al. (2005, 2007a, 2007b) summarized the stress intensity factor for anti-plane crack and two-dimension plane crack dynamic problems in functionally graded orthotropic medium. Guo et al. (2005) studied the dynamic response problem of an edge crack in a functionally graded orthotropic strip. It is apparent that the 3-D

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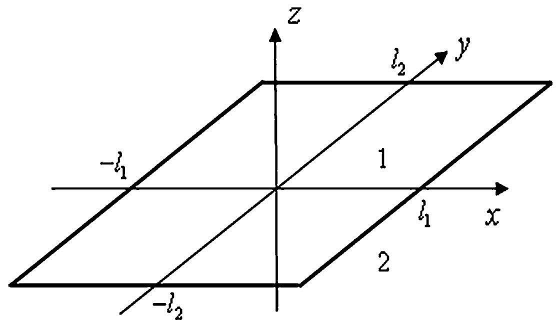

Fig. 1. Geometry and coordinate system for a rectangular crack.

crack problem in orthotropic material is not richer than 2-D crack problem because the complexity of mathematical derivation. However, the rectangular crack is a special case in the fracture mechanics. On the other hand, if an ellipse-like crack is replaced by a rectangular crack, the values of the stress intensity factors are larger than those for an ellipse-like crack (Itou, 1999). Therefore, the safety of the cracked structural member will be sufficiently ensured by comparing the fracture toughness value of the material with the stress intensity factors for a rectangular crack. The solution of the rectangular crack should be given for the theoretical studies of crack problem in an orthotropic elastic material.

It is the objective of the present work to provide a theoretical analysis of a rectangular crack in an orthotropic elastic material subjected to normal stress loading. Meanwhile, the generalized Almansi's theorem (Ding et al., 1996; Yang, 2001; Chen et al., 2004) and the Schmidt method (Morse and Feshbach, 1958; Yau, 1967) have not yet been used to study the behavior of a rectangular crack in a 3-D infinite orthotropic elastic material in the open literature. It is with this in mind that we report the present work. The process and method in the present paper are quite different from those literatures as mentioned above (Pan and Yuan, 2000; Wang et al., 2001; Liu et al., 2001; Wang and Shen, 2002; Chen et al., 2004; Zhao et al., 2007; Zhu and Qin, 2007).

2. Formulation of the problem

Consider an orthotropic elastic material with a symmetric rectangular crack located at $z = 0$ along the x-axis from $-l_1$ to l_1 and along the y-axis from $-l_2$ to l_2, as shown in Fig. 1. A Cartesian coordinate system (x, y, z) are assumed to coincide with the axes of elastic symmetry of the material in Fig. 1. It is assumed that a uniform normal traction $\sigma_{zz}(x, y, 0) = -\sigma_0$ (here σ_0 is the magnitude of the uniform tension stress loading) is directly applied on the crack surfaces, which is equivalent to investigate the perturbation fields for a remotely loaded cracked-body through the standard superposition technique in fracture mechanics. Therefore, the boundary conditions can be written as

$$\begin{cases} \sigma_{zz}^{(1)}(x, y, 0) = \sigma_{zz}^{(2)}(x, y, 0) = -\sigma_0 \\ \sigma_{xz}^{(1)}(x, y, 0) = \sigma_{xz}^{(2)}(x, y, 0) = 0 \qquad \text{for} \quad |x| \le l_1, \ |y| \le l_2 \\ \sigma_{yz}^{(1)}(x, y, 0) = \sigma_{yz}^{(2)}(x, y, 0) = 0 \end{cases} \tag{1}$$

$$\begin{cases} \sigma_{zz}^{(1)}(x, y, 0) = \sigma_{zz}^{(2)}(x, y, 0), \quad \sigma_{xz}^{(1)}(x, y, 0) = \sigma_{xz}^{(2)}(x, y, 0) \\ \sigma_{yz}^{(1)}(x, y, 0) = \sigma_{yz}^{(2)}(x, y, 0), \quad u^{(1)}(x, y, 0) = u^{(2)}(x, y, 0) \qquad \text{for} \quad |x| > l_1, \ |y| > l_2 \\ v^{(1)}(x, y, 0) = v^{(2)}(x, y, 0), \quad w^{(1)}(x, y, 0) = w^{(2)}(x, y, 0) \end{cases} \tag{2}$$

$$u^{(j)}(x, y, z) = v^{(j)}(x, y, z) = w^{(j)}(x, y, z) = 0 \quad \text{for} \quad \sqrt{x^2 + y^2 + z^2} \to \infty \tag{3}$$

where $\sigma_{zz}^{(j)}(x, y, z)$ is the z-component of stress tensor in the corresponding material, $\sigma_{xz}^{(j)}(x, y, z)$ and $\sigma_{yz}^{(j)}(x, y, z)$ are the x-component and y-component of stresses tensor in the corresponding material, respectively; $u^{(j)}(x, y, z)$, $v^{(j)}(x, y, z)$ and $w^{(j)}(x, y, z)$ represent the displacement components in the x-, y- and z-axis directions. The superscript j ($j = 1, 2$) denotes the fields in the upper half space 1 and the lower half space 2 as shown in Fig. 1, respectively.

3. Basic equations of the orthotropic elastic material

In Cartesian coordinates (x, y, z), the basic equations of linear, homogeneous, 3-D orthotropic elastic material ignoring body forces are

$$\sigma_{ik,k}^{(j)} = 0, \quad (i, k = x, y, z) \tag{4}$$

$$\sigma_{is}^{(j)} = \frac{c_{iskl}(u_{k,l}^{(j)} + u_{l,k}^{(j)})}{2}, \quad (i, s, l, k = x, y, z) \tag{5}$$

where c_{iskl} are the elastic stiffness constants. A subscript comma denotes the partial differentiation with respect to the coordinate. For an orthotropic elastic material, there are only nine independent elastic constants c_{11}, c_{12}, c_{13}, c_{22}, c_{23}, c_{33}, c_{44}, c_{55} and c_{66}.