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Electromechanical control of the dynamics of a thin elasticplate: Analytical method and finite differences simulation

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ABSTRACT

The electromechanical control of the dynamics of thin elastic plate is analysed using both the modal approach and the direct numerical simulation of partial differential equation. The electromechanical controller is constituted of a RL circuit with some stings connected through a magnet to the plate. The direct numerical simulation reveals the existence of different vibration modes. The boundary limits of the control parameters leading to the reduction of vibration amplitude, snap through instability and Melnikov chaos are determined and plotted in terms of the system parameters. It is seen that electromechanical control can eliminate the chaotic domain leading to periodic oscillations.

1. Introduction

Many mechanical and civil structures like bridges and buildings are subjected to vibrations from various sources: rotating engines, high-speeding cars and many other natural disturbances (earthquakes). These vibrations are responsible for the fatigue and damage which can result in the reduction or even loss of the performance of the structure. Thus many researchers have been interested by the control of linear and non-linear vibrating structures (Soong, 1950; Fuller et al., 1997; Aida et al., 1995, 1992; Nana Nbendjo et al., 2003).

In the literature many methods to reduce the vibration have been studied. Ashour and Nayfeh (2002) considered a non-linear adaptive control of flexible structures using the saturation phenomenon. They showed experimentally that the frequency-measurement technique is very efficient and the response of the beam is greatly reduced and saturated at a small value of the energy from the excitation source. Another method is to use the transduction mechanisms to reduce the vibrations. Hence Kitio et al. (2006) analysed the electromechanical control of the beam dynamics. They derived the critical parameters for the reduction of amplitude, for the control of snap-through instability and for the control of chaos using approximate analytical treatments and confirmed by the direct numerical simulation of the partial differential equation. Recently, Nanha et al. (2013) dealt with the enhancement of electromechanical control of vibration on a thin plate submitted to non-ideal excitation. They used Routh–Hurwitz criteria to obtain the stability condition of the controlled system and some dynamics exploration leading us to the condition for which the amplitude of vibration is reduced in the mechanical structure.

Apart the study of Kitio et al. (2006), most of the theoretical studies on vibration control of flexible structures are implemented by using the modal equations (Nana Nbendjo, 2009; Nanha et al., 2013). These equations are obtained when one applies approximation methods such as Galerkin method or Fourier series. Consequently the modal equations are an approximation of partial differential equations.

This work is an extension of the work by Kitio et al. (2006) to the case of a plate. The control of the dynamical behaviour of a plate under the action of an external periodic load is considered using both the modal approach and the direct numerical simulation of the partial differential equations describing the plate dynamics. This work is organized as follows. In Section 2 we present the physical system, the modal equation and the discretization scheme of the partial differential equation of the plate under control. In Section 3 we use the harmonic balance to obtain the good control parameters (linear case) and space parameters (non-linear) of the system to

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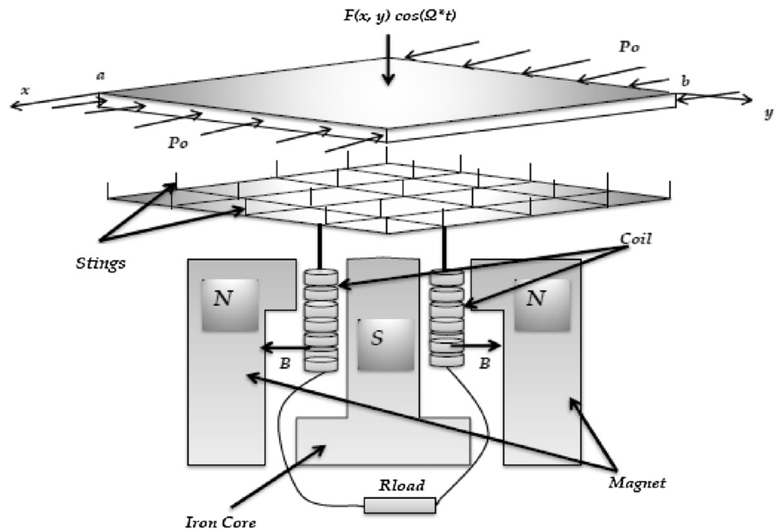

Fig. 1. Plate under electromechanical control.

predict the reduction of amplitude. In each case, the result of the simulation of the modal equations and the numerical simulation of the partial differential equations are given. Section 4 analyses the appearance of snap through instability and chaos and their control. Section 5 summarizes the work.

2. Physical system, modal equations and numerical scheme

2.1. Physical system

Based on the research presented by Shivamoggi (1997), we consider a rectangular thin elastic plate of constant thickness h, simply supported along the edges and subjected to a localized transverse periodic excitation and a compressive stationary load P_o per unit of length at the edges $x=0$ and a as shown in Fig. 1. For this works, let us consider an example of steel plate having the following parameters $E=2.11\times10^{11}$ N/m, ρ=7850 kg/m^3, a=0.8 m, b=1.2 m, $h=2\times10^{-3}$ m and λ=20 N s/m. We confine our attention to a middle plane, so that the coplanar displacement components u and v can be ignored. P_o acts in the middle plane of the plate. An electromechanical device (Nanha et al., 2013) composed by a RL circuit with some stings which represents the positions of the localized control device is connected to the plate as controller. A schematic of the set-up is shown in Fig. 1. The stings are regularly spaced and directly connected to the plate. The structure is sketched so as to clearly show the stings (there is no air gap between the stings and the main plate). Taking into account the electromechanical control device, the equations can be written as follows:

$$\rho h\frac{\partial^2 w}{\partial t^2}+\lambda\frac{\partial w}{\partial t}+D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\,\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)-\frac{Eh}{(1-\nu^2)}\frac{\partial^2 w}{\partial x^2}\times\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}\nu\left(\frac{\partial w}{\partial y}\right)^2-\frac{(1-\nu^2)}{Eh}P_o\right]-2\frac{Eh}{(1+\nu)}\frac{\partial^2 w}{\partial x\,\partial y}\times\left[\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right]-\frac{Eh}{(1-\nu^2)}\frac{\partial^2 w}{\partial^2 y}\left[\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{1}{2}\nu\left(\frac{\partial w}{\partial x}\right)^2\right]=F_o\cos(\omega t)\delta(x-x_o)\delta(y-y_o)-\frac{L_f BI}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M}\delta(x-x_i)\delta(y-y_j)\quad(1a)$$

$$L\frac{dI}{dt}+RI=L_f B\frac{\partial w}{\partial t}\sum_{i=1}^{N}\sum_{j=1}^{M}\delta(x-x_i)\delta(y-y_j)\quad(1b)$$

In Eq. (1) ρ represents the density of the thin plate, $D=Eh^3/12(1-\nu^2)$ is the bending rigidity, E is Young's modulus, ν is the Poisson ratio, λ is the damping coefficient, N and M are, respectively, the number of stings acting in x and y directions, respectively.

In order to obtain the dimensionless equations, we introduce the following transformations of the variables and parameters:

$\tau=\omega t$; $\omega^2=\left[\left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2\right]\sqrt{\frac{D}{\rho h}}$; $\lambda_1=\frac{\lambda}{\rho h\omega}$; $F_1=\frac{F_0}{\rho h\omega^2}$; $G=\left[\left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2\right]^2$; $G_1=\frac{E}{\rho\omega^2(1-\nu^2)}$; $P_{cr}=D\left(\frac{a}{\pi}\right)^2\left[\left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2\right]^2$; $G_2=\frac{2E}{\rho\omega^2(1+\nu)}$; $p_o=\frac{P_o}{P_{cr}}\left(\frac{a}{\pi}\right)^2$; $\alpha=\frac{L_f B}{\rho h\omega^2 NM}$; $\gamma=\frac{L_f B}{L}$; $\beta=\frac{R}{L\omega}$

Eqs (1a and 1b) reduce to

$$\frac{\partial^2 w}{\partial\tau^2}+\lambda_1\frac{\partial w}{\partial\tau}+\frac{1}{G}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+p_o\frac{\partial^2 w}{\partial x^2}-G1\frac{\partial^2 w}{\partial^2 x}\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}\nu\left(\frac{\partial w}{\partial y}\right)^2\right]-G1\frac{\partial^2 w}{\partial^2 y}\left[\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{1}{2}\nu\left(\frac{\partial w}{\partial x}\right)^2\right]-G2\frac{\partial^2 w}{\partial x\partial y}\left[\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right]=F_1\cos(\Omega\tau)\delta(x-x_0)\delta(y-y_0)-\alpha\sum_{io=1}^{N}\sum_{jo=1}^{M}\delta(x-x_{io})\delta(y-y_{jo})\quad(2a)$$

$$\frac{dI}{d\tau}+\beta I=\gamma\sum_{io=1}^{N}\sum_{jo=1}^{M}\frac{\partial w}{\partial\tau}\delta(x-x_{io})\delta(y-y_{jo})\quad(2b)$$