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Frequency-dependent, near-pole behavior of acoustic surface waves on a solid sphere

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ABSTRACT

A laser vibrometer is used to measure waves generated by a piezoelectric transducer in a solid steel sphere. The spatial and temporal resolution make it possible to verify the theoretical dispersion relation of spherical surface waves. In particular, the area at the source antipode is investigated. The focusing of surface waves results in high-amplitude surface displacements in this area, which is therefore interesting in terms of seismic research and the understanding of the dynamic behavior of granular media. The influence of frequency on the diameter of the focal area is measured using windowed sine bursts as a source. As an example of the complex wave behavior near the poles, the interference of converging and diverging surface waves is demonstrated.

1. Introduction

The behavior of elastic waves in spherical bodies gained interest for the study of seismic phenomena (Sato and Usami, 1962; Rial and Cormier, 1980; Bolt and Dorman, 1961). More recently, waves in spherical bodies are being investigated for their contribution to the dynamics of granular solids (De Billy, 2000; Anfosso and Gibiat, 2004; Van Damme and Spadoni, 2013), and for applications such as acoustophoresis (Leibacher et al., 2013), gas sensors (Nakamoto et al., 2008; Yamanaka et al., 2009), and optomechanics (Zehnpfennig et al., 2011; Matsko et al., 2012). Spherical waves on the surface of a sphere in particular can be used for non-destructive testing of bearing balls (Yamanaka et al., 2000), the development of chemical sensors (Nakamoto et al., 2008; Yamanaka et al., 2009) and optoacoustic-coupling devices capable of generating RF waves (Matsko et al., 2012). Mechanical wave propagation in granular media is also affected by surface waves; their spectral signature has been detected in the response to high-frequency excitation (De Billy, 2000; Anfosso and Gibiat, 2004) and it has also been linked to the low-frequency response of a 1D chain of spheres excited by an impact (Van Damme and Spadoni, 2013). All these applications are concerned with the characteristics of surface waves, but do not emphasize the convergence of surface waves over spheroidal bodies into narrow, high-amplitude waveforms. This is relevant for example for nano-printing technology, where ink nano-droplets are pinched off from larger droplets (Galliker et al., 2012).

The theoretical dynamic response of a point source on the surface of an elastic sphere has been derived in Lamb (1881). This solution however has a singularity at the source antipode (Clorennec and Royer, 2004). Among the three predicted wave types generated by a force exerted on the surface moreover, surface or Rayleigh waves are of particular interest since their amplitude is significantly higher than the amplitude of bulk waves (longitudinal and shear) (Ishikawa et al., 2003). This is in line with the theoretical energy partition among wave modes in a semi-infinite medium excited by a harmonic point-load, as described in Miller and Pursey (1955), where they found that 67% of the wave energy created by a sinusoidal force on the surface is carried by surface waves. These waves are useful to estimate graded material properties (Chiriţă, 2013) and have some remarkable properties, studied both theoretically and experimentally. A first well-studied feature of spherical surface waves is their dispersive behavior due to the curvature of the surface (Sato and Usami, 1962; Viktorov, 1967). This has been experimentally validated in Otsuka et al. (2010). Second, surface waves favour certain frequencies due to the cyclic symmetry of the wave guide and stress boundary-conditions which lead to a discrete spectrum (Rayleigh, 1910; Zehnpfennig et al., 2011; Viktorov, 1967; Lapwood and Usami, 1981). Finally, surface waves travelling across the source antipode undergo a $\pi/2$ phase shift (Brune et al., 1961), and their experiments show a signal that is much more complex in the poles than on any other point of the surface.

Most of the experimental work carried out so far has focused on determining the wave (phase or group) velocity and the

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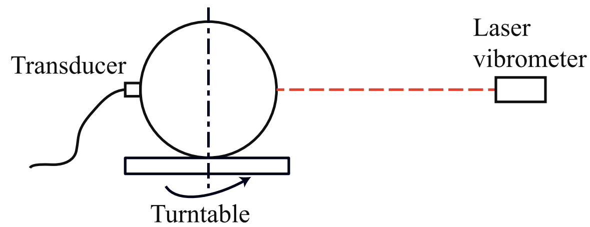

Fig. 1. Setup of the experiments. A sphere is positioned on a precision turntable to allow the laser vibrometer to scan a line along a great circle centered around the rotation axis.

amplitude variation away from poles, usually by using a short excitation source generated by a pulsed laser (Clorennec and Royer, 2004; Otsuka et al., 2010; Tsukahara et al., 2000). This approach however is well suited for high-frequency spectra given the pulse duration is in the ns range, while the aim of our investigations is the behavior of surface waves in the kHz range. In previous studies furthermore, the area near the source antipode has mostly been ignored because of the complexity of the local wavefield. Seismological studies have shown however that a small area around the antipode provides significant information about the various wave types and reflections in a sphere due to geometric focusing (Rial and Cormier, 1980; Chael and Anderson, 1982).

This paper presents an experimental technique to investigate all of the aforementioned properties of spherical surface waves. The proposed experiments provide flexibility regarding amplitude and frequency of the wave source and both spatial and temporal-measurement accuracy. The use of windowed, sinusoidal bursts allows for the investigation of frequency dependence of these phenomena. The measured dispersion, frequency content and frequency-dependent focusing are compared to theoretical models predicting the dynamic response for the excitation signals we used. The focusing of surface waves at the antipode is of particular interest to investigate the complex wave behavior in this area.

2. Materials and methods

The setup of the experiments consists of a 100 mm-diameter stainless steel sphere (AISI 316), resting on a rotating table with a 0.005° rotation angle precision (Fig. 1). The turntable contact zone is much smaller than any wavelength of surface waves and it also coincides with the location with lowest amplitude of surface waves. Piezo actuators are used as an acoustic source. For the low frequency ultrasound range (50–150 kHz), a small piezostack is used (Piezomechanik 2 mm× 3 mm × 5 mm). Higher frequencies (up to 350 kHz) can be achieved by a piezoelectric transducer accommodated in a steel casing with a 6 mm diameter (DAKEL midi). The transducers are driven by a Tektronix AFG 3022C arbitrary waveform generator. The source signals are short sinusoidal bursts (1–3 periods, depending on the frequency), modulated by a hamming window to limit the frequency content. The generated waves are measured by a Polytec OFV-534 laser vibrometer and recorded with a Tektronix DPO 3012 digital oscilloscope. The source and the laser are aligned so that a great circle through the source and its opposite pole can be scanned by rotating the sphere. The advantage of this arrangement over previously described experiments is the absence of additional mass altering boundary conditions at the antipode of the source. No modifications of the surface, like a thin film (Yamanaka et al., 2009; Otsuka et al., 2010) or a measuring transducer (Tsukahara et al., 2000) are required. Moreover, the investigated surface needs no special treatment. However, multiple surface-wave roundtrips are disturbed by piezoelectric transducers used for excitation. These may induce mode conversion at the excitation location, reducing amplitude, but still allow an analysis of the surface-wave frequency content.

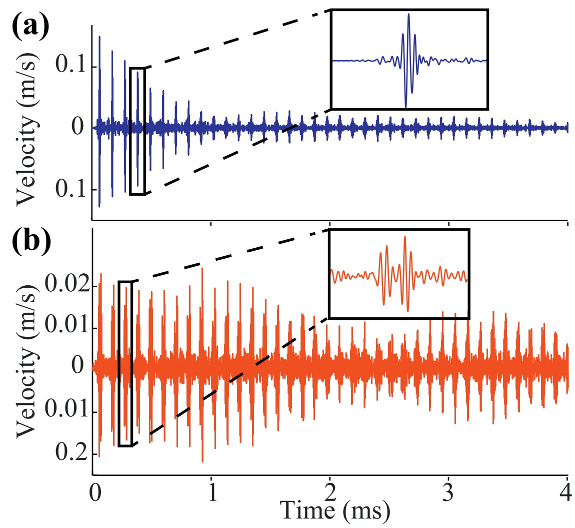

Fig. 2. Time histories of signals measured in the antipode of an acoustic source generating a short sinusoidal burst (a) and at 25° away from the antipode (b). The source signal is a 3-period 206 kHz hanning-windowed sinusoidal burst. The periodic arrivals of Rayleigh wave pulses are clearly visible.

3. Results and discussion

A typical response signal resulting from a short sinusoidal burst is shown in Fig. 2 at the antipode (a) and away from the antipode (b). The high-amplitude pulses are surface waves arriving with regular intervals defined by their wave speed and the circumference of the sphere. For a 100 mm-diameter stainless-steel sphere and a pulse with central frequency 206 kHz, this interval is measured to be 107 μs.

The measured surface waves can be visualized as a ring with the pole-to-pole line as an axis, with a radius and a particle amplitude that vary as the wave travels over the sphere. Due to conservation of energy, the amplitude is lowest at the equator and highest at the poles, as the ring focuses theoretically in a single point (Viktorov, 1967). Although viscous damping and dispersion of the pulses make the amplitude decay, up to 50 roundtrips can be measured. The amplitude is much lower for signals away from the antipode, as is shown in Fig. 2(b). At 25° away from the pole, the measured velocity is reduced almost by a factor 10. Each high amplitude peak now consists of two minor peaks. The first is the signal still converging towards the pole, whereas the second already traveled over the pole and is diverging (Otsuka et al., 2010).

The authors first verified the dispersion relation and the surface wave frequencies of the considered sphere by measuring the time history of surface signals along a line with a 0.04° step over a total angle of 120°. The sampled time history of the surface velocity at each location is assembled into a matrix $\boldsymbol{s}(x_k, t_l)$, where x_k in the location of the k^{th} station and t_l is the l^{th} time step. A two-dimensional, discrete Fourier transform is employed to define the amplitude spectrum of measured signals:

$$M_{r,m} = \sum_{k=0}^{K-1}\sum_{l=0}^{L-1} s(x_k, t_l)e^{-i2\pi k/K/\lambda_m}e^{-i2\pi f_r l/L}, \tag{1}$$

where $i = \sqrt{-1}$, r, m are integers f_k and λ_m are the frequency and wavelength respectively; $K = 3000$ and $L = 16384$. The bandwidth of signals obtained with the laser vibrometer extends to 3 MHz. Eq. (1) is implemented numerically in Matlab via the function *fft2*. The