Mechanics Research Communications 60 (2014) 45–51

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Nonlinear free vibration of nanotube with small scale effects embedded in viscous matrix

Yi-Ze Wang[a,*], Feng-Ming Li[b,*]

[a] P. O. Box 137, School of Astronautics, Harbin Institute of Technology, Harbin 150001, PR China
[b] College of Mechanical Engineering, Beijing University of Technology, Beijing 100124, PR China

ARTICLE INFO

Article history:
Received 9 September 2013
Received in revised form 15 April 2014
Accepted 8 June 2014
Available online 14 June 2014

Keywords:
Nonlinear vibration
Small scale effects
Nanotube
Viscous elastic matrix
Damping

ABSTRACT

In this paper, the nonlinear free vibration of the nanotube with damping effects is studied. Based on the nonlocal elastic theory and Hamilton principle, the governing equation of the nonlinear free vibration for the nanotube is obtained. The Galerkin method is employed to reduce the nonlinear equation with the integral and partial differential characteristics into a nonlinear ordinary differential equation. Then the relation is solved by the multiple scale method and the approximate analytical solution is derived. The nonlinear vibration behaviors are discussed with the effects of damping, elastic matrix stiffness, small scales and initial displacements. From the results, it can be observed that the nonlinear vibration can be reduced by the matrix damping. The elastic matrix stiffness has significant influences on the nonlinear vibration properties. The nonlinear behaviors can be changed by the small scale effects, especially for the structure with large initial displacement.

1. Introductions

Carbon nanotube possesses the excellent electrical, chemical, thermal and mechanical characteristics (Iijima, 1991; Lau et al., 2006; Spitalsky et al., 2010). With these superior properties, nano systems have shown various potential applications, such as atomic-force microscope, composite nanofibers, nanobearings and nanoactuators, etc. As a result, a lot of work has been carried out on the mechanical properties of nanotubes with both theoretical and experimental methods (Gibson et al., 2007; Sun et al., 2009; Shokrieh and Rafiee, 2010; Araujo dos Santos, 2011). Because it is difficult to control the experiment at the nano scale, numerical simulation and analysis have been performed widely.

Although the molecular dynamics (MD) simulation is an effective way to investigate the mechanical behaviors of nano structures, it is rather difficult for large-scale nano systems. Furthermore, with the characteristic of less time-consuming, the continuum model has been applied in many investigations (Yoon et al., 2006; Wang and Cai, 2006; Wang and Varadan, 2006; Pentaras and Elishakoff, 2011). However, the classical continuum method cannot illustrate the small scale effects which become more obvious and important for nano systems.

The nonlocal continuum theory presented by Eringen (1972, 1983) assumes that the stress at a reference point is a function of the strain at every point in the body. After the first several investigations on mechanical properties of nanotubes with the nonlocal continuum theory (Sudak, 2003; Peddieson et al., 2003), many researches have been reported on the characteristics of buckling (Li and Kardomateas, 2007; Amara et al., 2010; Pradhan and Reddy, 2011), vibration (Xia and Wang, 2010; Kiani and Mehri, 2010; Murmu and Adhikari, 2011; Lim and Yang, 2011; Ghavanloo et al., 2011; Lee and Chang, 2010; Simsek, 2010) and wave propagation (Lu et al., 2007; Wang et al., 2008; Li et al., 2008) of nanotubes. For more information about the nonlocal continuum theory applied to nano structures, one can refer to Refs. (Wang et al., 2010; Arash and Wang, 2012) and the references therein.

Different from a lot of work have been presented on the linear vibration behavior of nanotubes, investigations on the nonlinear properties are rather limited. Only several investigations on nonlinear problems with both classical (Fu et al., 2006; Yan et al., 2011; Ansari and Hemmatnezhad, 2012) and nonlocal (Yang and Lim, 2009; Ke et al., 2009; Yang et al., 2010; Reddy, 2010; Shen and Zhang, 2011; Fang et al., 2013; Simsek, 2014) continuum theories have been reported. Most of the above work is mainly concerned on the amplitude–frequency response of the nanotube. However, little work can be found for nonlinear vibration properties of nano systems with the damping effect. In the present paper, the nonlinear free vibration of the nanotube is studied by the nonlocal continuum theory. Both the damping and small scale effects are

* Corresponding authors. Tel.: +86 451 86414479 (Y.-Z. Wang), +86 10 67392704 (F.-M. Li).
E-mail addresses: wangyize@126.com (Y.-Z. Wang), fmli@bjut.edu.cn (F.-M. Li).

http://dx.doi.org/10.1016/j.mechrescom.2014.06.002
0093-6413/

considered and nonlinear vibration behaviors of the nanotube are illustrated.

2. Equations of nonlinear vibration

The nanotube embedded in the viscous elastic matrix is shown in Fig. 1. According to the work of Eringen (1972, 1983), the constitutive relation of nonlocal elasticity is presented with the form of the integral equation as

$$\sigma_{kl,\,k}-\rho\,\ddot{u}_l=0, \tag{1a}$$

$$\sigma_{kl}(\mathbf{x})=\int_V \alpha(\mathbf{x},\mathbf{x}')\tau_{kl}(\mathbf{x}')dV(\mathbf{x}'), \tag{1b}$$

$$\varepsilon_{kl}=\frac{1}{2}(u_{k,\,l}+u_{l,\,k}), \tag{1c}$$

where σ_{kl} is the nonlocal stress tensor, ε_{kl} the strain tensor, ρ the mass density, u_l the displacement vector, $\tau_{kl}(\mathbf{x}')$ the classical (i.e. local) stress tensor, $\alpha(\mathbf{x},\mathbf{x}')$ the kernel function which describes the influence of the strains at various location $\mathbf{x}'$ on the stress at a given location $\mathbf{x}$ and V the entire body considered.

We can observe from Eq. (1) that not only the strain state of the location $\mathbf{x}$ has the influence on the stress, but also the strain state at $\mathbf{x}'$ can affect on the stress state of the same location. Because it is difficult to use the constitutive relation with the integral forms, the partial differential expressions are derived and applied widely, which are presented as (Amara et al., 2010; Simsek, 2014; Chang, 2012):

$$[1-(e_0a)^2\nabla^2]\quad \boldsymbol{\sigma}=\mathbf{C}_0:\boldsymbol{\varepsilon}, \tag{2}$$

where $\mathbf{C}_0$ is the elastic stiffness matrix of the classical elasticity, $\boldsymbol{\varepsilon}$ the strain vector, e_0 the constant and a the internal characteristic length (e.g. the length of C—C bond, the lattice spacing and the granular distance). It should be noted that e_0a means the scale coefficient which denotes the small scale effect on the mechanical characteristics of nano structures. It will be reduced to the classical (i.e. local) model for $e_0a=0$.

For the one-dimensional stress state, the Hook's law with the nonlocal continuum theory can be expressed as the following form:

$$\sigma_x-(e_0a)^2\frac{\partial^2\sigma_x}{\partial x^2}=E\varepsilon_x, \tag{3}$$

where E is the Young's modulus.

For the Euler–Bernoulli beam model, the axial force and the resultant bending moment are

$$N=\int_A \sigma_x dA,\ M=\int_A z\sigma_x dA, \tag{4}$$

where A the area of the cross section for the nanotube.

The displacement fields can be expressed as the following form:

$$u_1(x,z,t)=u(x,t)-z\frac{\partial w}{\partial x},\quad u_2=0,\quad u_3(x,z,t)=w(x,t) \tag{5}$$

where u and w are the axial and transverse displacements, respectively.

For the nonlinear vibration with the large amplitude, the nonzero von Kármán nonlinear strain (i.e. ε_{non}) should be considered and the relation between the strain and displacement is

$$\varepsilon_0=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,\quad \varepsilon_1=-z\kappa, \tag{6}$$

where ε_0 is the nonlinear extensional strain, $\kappa=-\partial^2 w/\partial x^2$ the bending strain and ε_1 the strain induced by κ. Then the von Kármán nonlinear strain (i.e. ε_{non}) is

$$\varepsilon_{non}=\varepsilon_0+\varepsilon_1=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-z\frac{\partial^2 w}{\partial x^2}, \tag{7}$$

From Eqs. (3)–(7), the following relation can be derived:

$$N-(e_0a)^2\frac{\partial^2 N}{\partial x^2}=EA\varepsilon_0, \tag{8a}$$

$$M-(e_0a)^2\frac{\partial^2 M}{\partial x^2}=EI\kappa, \tag{8b}$$

where $I=\int_A z^2dA$ is the moment of inertia.

The kinetic energy T can be expressed as the following form:

$$T=\frac{1}{2}\rho A\int_0^L\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]\mathrm{d}x. \tag{9}$$

Moreover, for the elastic matrix described as the Winkler model with the viscous damping, the pressure caused by the elastic stiffness of per unit axial length is $-k_w\times w$ and k_w denotes the matrix material constant.

The strain energy U is

$$U=\frac{1}{2}\int_0^L\int_A(\sigma_{ij}\varepsilon_{ij})\,\mathrm{d}A\,\mathrm{d}x. \tag{10}$$

According to Eqs. (7) and (10), we can derive that expression of the strain energy as

$$U=\frac{1}{2}\int_0^L\left\{N\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]-M\frac{\partial^2 w}{\partial x^2}\right\}\mathrm{d}x. \tag{11}$$

Moreover, the virtual work by the external load from the viscous elastic matrix can be written as

$$\delta W_{ext}=\int_0^L q\delta w\,\mathrm{d}x, \tag{12}$$

where $q=-[k_w w+c(\partial w/\partial t)]$ is the load exerted by the viscous elastic foundation and c the damping coefficient.

It has shown that the stick-slip mechanism at the interface of the carbon nanotubes and surrounding matrix is a likely cause of damping in materials with embedded nanotubes (Zhou et al., 2004; Dai and Liao, 2009; Johnson et al., 2011; Liu et al., 2010, 2011). More and deeper understanding about the damping mechanism between the nanotube and matrix is still an open research subject. But it should be noted that in the present work, the nonlocal beam model is applied to simulate the nonlinear vibration behaviors of the nanotube. Then the damping coefficient is added to describe the damping effect of the matrix. Because in the classical publication, the nonlinear vibration of the classical beam model with the damping effect has been presented by this method (Nayfeh and Mook, 1979), although the damping term in Eq. (10) cannot be simply considered as the inclusion or exclusion from the stick-slip mechanism, such simple and reliable model has been accepted. Moreover, this analysis is used and has shown its feasibility on nonlinear vibration of the nanotube with the classical (i.e. local) beam model (Rasekh et al., 2010).

From the Hamilton's principle, we have the following relation:

$$\int_0^t \delta[T-(U-W_{ext})]\mathrm{d}t=0. \tag{13}$$