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On the dynamics of viscoelastic discontinuous beams

Giuseppe Failla *

Dipartimento di Ingegneria Civile, dell'Ambiente, dell'Energia e dei Materiali (DICEAM), Università di Reggio Calabria, Italy

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ABSTRACT

This paper concerns the dynamics of beams with an arbitrary number of Kelvin–Voigt viscoelastic rotational joints, translational supports, and attached lumped masses. Using the theory of generalized functions to treat the discontinuities of the response variables, the free vibration problem is solved upon deriving exact closed-form eigenfunctions, that inherently fulfill the required conditions at the discontinuity points. The forced vibration response is computed in time and frequency domain by modal superposition, based on appropriate orthogonality conditions of the eigenfunctions.

1. Introduction

Engineering applications frequently involve beams with joints, supports and attachments. Within a standard 1D formulation of the governing equations, beams of this sort are commonly referred to as *discontinuous* beams, because pertinent discontinuities have to be considered in the response variables. Especially in structural dynamics, rotational joints (RJs), translational supports (TSs) and attached lumped mass (LMs) play an important role and, in many cases, both flexibility and damping shall be accounted for in RJs and TSs. In this context, a typical constitutive law of RJs and TSs involves linear elasticity and viscous damping, corresponding to a Kelvin–Voigt (KV) viscoelastic law. Examples are in TSs modeling external dampers for vibration control (Shukla and Datta, 1999; FEMA 273), RJs modeling dissipating beam-to-column connections (Xu and Zhang, 2001), RJs modeling bolted or welded joints with imperfections or damage (Kawashima and Fujimoto, 1984; Sekulovic et al., 2002).

A classical solution to the eigenvalue problem of beams with KV viscoelastic RJs, TSs and LMs involves expressing, over uniform segments between consecutive discontinuity points, the vibration response in a trigonometric form with four unknown constants, and enforcing matching conditions at the discontinuity points along with the boundary conditions (B.C.). In this context, a complex modal analysis is generally required, being the damping non-proportional (Veletsos and Ventura, 1986). A similar approach should be pursued to compute the frequency response function (FRF). However, the equations rapidly increase with the number of discontinuities, and must be updated whenever their locations change along the beam axis. These are significant drawbacks for design, optimization and sensitivity analyses, which require building different solutions for different potential system parameters. Motivated by these issues, alternative solutions have been sought.

Approximate solutions have been obtained by an assumed-mode method based on eigenfunctions of the *bare* beam, in conjunction with a variational formulation. In this context, the characteristic equation (CE) has been built as determinant of an appropriate matrix by Cha (2005) for a EB beam with multiple KV viscoelastic TSs, and has been derived in a closed form by Gürgöze (1998) for a EB cantilever beam carrying a tip mass and supported by an in-span damper. Also, Gürgöze and Erol (2002) have built an approximate time domain response function for an EB cantilever beam carrying a tip mass, with an in-span damper and a fixed support.

In some cases exact solutions have been also found. For instance, an exact closed-form CE has been derived by Chang et al. (2001) based on the Laplace transform, for a EB carrying an arbitrary number of elastic or viscous TSs and LMs. Following a transfer matrix approach, Sorrentino et al. (2003) have built the CE as determinant of a 4×4 matrix, regardless of the number of discontinuity points, the FRF and the impulse response function (IRF) by modal superposition upon demonstrating orthogonality conditions of the eigenfunctions, for KV

* Tel.: +39 0965 875221; fax: +39 0965 875201.
E-mail address: giuseppe.failla@unirc.it

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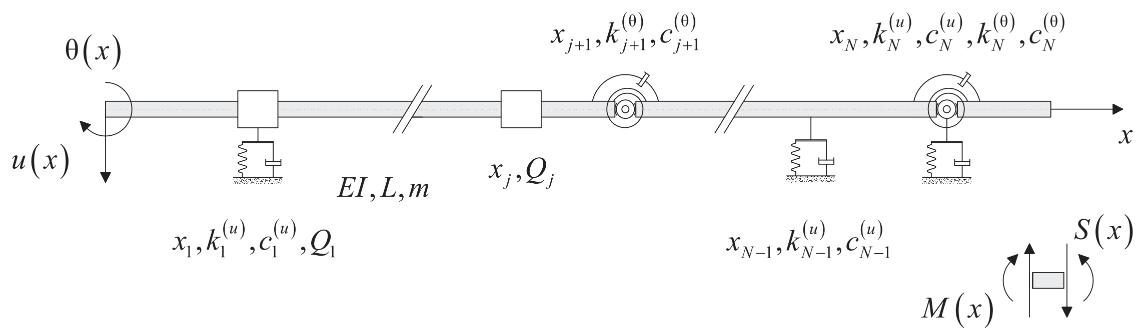

Fig. 1. Beam with arbitrary number of RJs, TSs and LMs.

viscoelastic TSs and LMs in either EB or Timoshenko (TM) beams (Sorrentino et al., 2003, 2007). Following a dynamic stiffness matrix approach, Hong and Kim (1999) have built the CE as determinant of a beam dynamic stiffness matrix obtained by assembling those of the uniform segments between two consecutive discontinuity points, and a closed-form FRF matrix and a IRF matrix by modal superposition, for TM beams with KV viscoelastic RJs, TSs and LMs (Hong and Kim, 1999). However, by this approach the size of the matrices inevitably increases with the number of discontinuity points.

With the aim of contributing to the research effort in this field, this paper will focus on EB beams with an arbitrary number of KV viscoelastic RJs, TSs and LMs. Using the theory of generalized functions to treat the discontinuities of the response variables (Caddemi and Caliò, 2009, 2013; Caddemi et al., 2013a; Failla and Santini, 2007, 2008; Failla, 2011; Failla and Impollonia, 2012), a solution will be found to free and forced vibrations. In particular, the exact CE will be obtained as determinant of a 4×4 matrix, upon deriving exact closed-form expressions for the eigenfunctions, that inherently fulfill the required conditions at the discontinuity points. Hence, orthogonality conditions of the eigenfunctions, closed-form modal impulse response functions (MIRFs), and closed-form modal frequency response functions (MFRFs) will be obtained generalizing a theoretical framework previously devised by Oliveto et al. (1997) for a EB beam with viscous RJs at the ends. The MIRFs and MFRFs will be used to solve forced vibrations problems, using the complex modal superposition principle as presented by Oliveto et al. (1997).

2. Free vibration equations by theory of generalized functions

Consider the EB beam in Fig. 1. Be x the longitudinal axis (positive rightward), L the length, EI the flexural stiffness, m the mass per unit length. Denote by $u(x)$ and $\theta(x)$ deflection and rotation of the cross section, by $M(x)$ and $S(x)$ bending moment and shear force. Positive signs are shown in Fig. 1.

It is assumed that an arbitrary number of KV viscoelastic RJs, TSs, LMs are arbitrarily located at abscissas x_j's along the beam axis, for $j = 1, 2, \ldots N$, $0 < x_1 < \cdots < x_j < \cdots < x_N < L$; x_j's will be referred to as discontinuity points. Stiffness and damping parameters are $k_j^{(\theta)}$ and $c_j^{(\theta)}$ for the jth RJ, $k_j^{(u)}$ and $c_j^{(u)}$ for the jth TS; Q_j is the jth LM. Equations will be written for the most general case when a RJ, a TS and a LM occur simultaneously at every discontinuity point. At any rate, straightforward changes can be made, whereas any of the latter is not present at a discontinuity point. Following the approach by Wang and Qiao (2007), who have studied EB beams with discontinuities due to linearly-elastic RJs, TSs, LMs and other type of discontinuities, the free vibration equilibrium equations read

$$\frac{\bar{\partial} S}{\partial x} + \sum_{j=1}^{N} V_j \delta(x - x_j) = m\ddot{u}(x); \tag{1}$$

$$\frac{\bar{\partial} M}{\partial x} = S(x) \tag{2}$$

where dot means derivative with respect to time and bar means generalized derivative; $\delta(x - x_j)$ is the Dirac's delta, given as the generalized derivative of the unit-step function $H(x - x_j)$ (Falsone, 2002)

$$\delta(x - x_j) = \frac{\bar{\partial} H(x - x_j)}{\partial x}; \tag{3}$$

$$H(x - x_j) = 0, \quad x < x_j; \qquad H(x - x_j) = 1, \quad x > x_j. \tag{4a,b}$$

Also, in Eq. (1) V_j is the shear-force discontinuity due to the TS and the LM at x_j, i.e.

$$V_j = V_j^{(u)} + V_j^{(Q)}, \tag{5}$$

$$\text{for} \quad V_j^{(u)} = -k_j^{(u)} u(x_j) - c_j^{(u)} \dot{u}(x_j); \quad V_j^{(Q)} = -Q_j \ddot{u}(x_j), \tag{6a,b}$$

being $V_j^{(u)}$ and $V_j^{(Q)}$ both positive downwards. For $u(x)$ and $\theta(x)$, it can be written that

$$\frac{\bar{\partial} \theta}{\partial x} = -\frac{M(x)}{\text{EI}} + \sum_{j=1}^{N} r_j \delta(x - x_j); \tag{7}$$