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Micro-inertia effects on the dynamic characteristics of micro-beams considering the couple stress theory

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ABSTRACT

This paper presents a new model for the free transverse vibrations of an Euler–Bernoulli beam using the couple stress theory of elasticity with micro-structure. Introducing the kinematic variables, the strain and kinetic energy expressions (involving micro-inertia effect) have been obtained and the Hamilton principle has been used to derive the governing equations and the related boundary conditions of the free vibrations of fixed–fixed and simply supported beams. A numerical solution has been used to study the natural frequencies, mode shapes and free vibrations of the beams. A comparative result has shown that the bending rigidity predicted by the couple stress, is closer to the experiment result than that predicted by the modified couple stress theory. The results have shown that the bending rigidity of the beams depends on the ratio of the length scale to the beam thickness, whereas the micro-inertia term depends on the ratio of the length scale to the beam length.

1. Introduction

Beams and plates are key components of many engineering structures from nano to macro scales. There are several theories that deal with the mechanical behavior of these components. In the classical theory of linear elasticity, the material is considered as a continuum in mathematical sense. In such continuum the atomic structure of the material is neglected and the material particle is considered simply a geometrical point. This theory is inadequate for describing the mechanical behavior of materials with microstructure, such as polymeric foams, high-toughness ceramics, high strength metal alloys, granular materials or porous bones, because their behavior is characterized by non-local stresses and the existence of an internal length scale. Micro-structural effects are also important when structures have extremely small overall dimensions, which are comparable to the internal length scale of their material (Papargyri-Beskou and Beskos, 2008). Voigt was the first who tried to correct these shortcomings of classical elasticity by taking into account the assumption that interaction between the two parts through an area element inside the body is transmitted not only by a force vector but also by a moment vector giving rise to a 'couple stress theory' (Voigt, 1887). The complete theory of asymmetric elasticity was developed by Cosserat and Cosserat (1909), which was non-linear in the beginning. They assumed that each material point of a three dimensional continuum is associated with a 'rigid triad' and during the process of deformation; it can rotate independently in addition to the displacement. After a gap of about fifty years, Cosserats theory drew attention of researchers and several Cosserat-type theories were developed independently (e.g., Aero and Kuvshinskii, 1960; Eringen, 1962; Grioli, 1960; Gunther, 1958; Koiter, 1964; Mindlin and Tiersten, 1962; Nowacki, 1974; Palmov, 1964; Rajagopal, 1960; Toupin, 1962), among several others. Later, the general Cosserat continuum theory acquired the name of 'micropolar continuum theory' following Eringen (1966a,b), in which the micro-rotation vector is taken independent of displacement vector. Eringen and Suhubi (1964) and Suhubi and Eringen (1964) developed a non-linear theory for 'micro-elasticity', in which intrinsic motions of the microelements were taken into account. A further generalization of the continuum with microstructure leads to micromorphic continuum of Eringen (1966a,b). Micromorphic continuum treats a material body as a continuous collection of a large number of deformable particles, with each particle possessing finite size and inner structure. Using assumptions such as infinitesimal deformation and slow motion, micromorphic theory can be reduced to microstructure theory of Mindlin (1964). When the microstructure of the material is considered rigid, it becomes the Eringen's micropolar theory (Eringen, 1966a,b). Assuming a constant micro-inertia, Eringen's micropolar theory is identical to the Cosserats theory. Eliminating the distinction of macro-motion of the particle and the micro-motion of

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its inner structure, it becomes couple stress theory (Mindlin and Tiersten, 1962; Toupin, 1962). Moreover, when the particle reduces to a mass point, all theories reduce to classical or ordinary continuum mechanics.

The general theory of Mindlin includes three equivalent forms which are defined on the basis of three different expressions for the strain energy density. The first expression involves gradients of displacements, the second gradients of strain and the third gradients of rotation. The couple stress theory is based on this third expression of the strain energy density while second form leads to the gradient elastic theory.

The classical couple stress elasticity theory is a higher order continuum theory that contains two higher-order material length-scale parameters appear in addition to the two classical Lame constants. In this classical conception, only the conventional equilibrium relationships of forces and moments (of forces) are enforced and the couple is unconstrained in the absence of higher order equilibrium requirements. The couple stress theory has been applied to model the pure bending of a circular cylinder by Anthoine (2000). He has reported that the bending inertia of a circular cross-section results in higher values than those accepted before, especially when the ratio of the radius of the beam to the characteristic material length is lower than 20.

Yang et al. in 2002 introduced the modified couple stress theory. Beside the two conventional equilibrium relationships in the classical couple stress, they proposed an additional relation to constrain the couple. This relation considers the balance of moment of rotational momentum. This assumption make the couple stress tensor symmetric. Utilizing the modified couple stress theory, Park and Gao (2006) studied the static response of an Euler–Bernoulli beam and interpreted the outcomes of an epoxy polymeric beam bending test. Kong et al. (2008, 2009) derived the governing equation, initial and boundary conditions of an Euler–Bernoulli beam using the modified coupled stress theory and strain gradient elasticity theory. As they reported, the stiffness of beams is size-dependent. In addition, the difference between the stiffness obtained by the classical beam theory and those predicted by the modified couple stress theory is significant when the beam characteristic size is comparable to the internal material length-scale parameter. Recently, Fathalilou et al. (2011) have used the modified couple stress theory to study the pull-in instability of a gold micro-beam switch with the specifications introduced in the experimental work of Ballestra et al. (2010). As they reported, although using the modified couple stress theory leads to better results than the classic theory, yet there is a considerable difference between the results of the experiments and the modified couple stress theory.

Beam bending models based on other non-classical elasticity theories have also been reported. Papargyri-Beskou et al. (2003) have derived and solved the governing equation and corresponding boundary conditions of the beam buckling and bending using the simple strain gradient theory. Lazopoulos and Lazopoulos (2010) have studied the bending and buckling problem of thin beams using strain gradient theory with the terms depending upon the area of the cross-section of the beam.

In spite of mentioned studies about the mechanical behavior of Euler–Bernoulli beams using various elasticity theories, there is no comprehensive modeling of beams in the literature using the couple stress theory of elasticity with micro-structure. The objective of this paper is to introduce a non-classic model for the free vibrations of an Euler–Bernoulli beam using the concepts of the couple stress theory. The present model involves the micro-rotation effects leading to an added inertia in the dynamic motion. The governing equation and boundary conditions for the beams are obtained using the Hamilton principle. Unlike the existence of two non-classic material length scale in the couple stress theory for the general continuum, only one length scale parameter is appeared in the beam model. A comparative result shows that the bending rigidity predicted by the couple stress, is closer to the experiment than that predicted by the modified couple stress theory.

2. Fundamental equations of the couple stress theory

In the linear couple stress theory, the strain energy, in addition to the strain, is a function of the rotation-gradient (Mindlin, 1964). In the following sub-sections the helpful kinematic variables, strain and kinetic energies and constitutive equations of this theory are presented.

2.1. Kinematic variables

In the Cartesian coordinates, we define u_i to represent the displacement field of the continuum material. The displacement gradient tensor $u_{i,j}$ can be decomposed into symmetric and skew-symmetric parts as strain and rotation tensors, respectively:

$$u_{i,j} = \varepsilon_{ij} + \omega_{ij} \quad (i, j = 1, 2, 3) \tag{1}$$

where

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

$$\omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i}) \tag{3}$$

The rotation vector dual to the rotation tensor is defined as

$$\theta_i = \frac{1}{2}e_{ijk}\omega_{kj} = \frac{1}{2}e_{ijk}u_{k,j} \tag{4}$$

where e is the alternator.

The other kinematic variable to be taken into account in this theory is the gradient of rotation:

$$\kappa_{ij} = \theta_{j,i} = \frac{1}{2}e_{jkl}\omega_{kl,i} \tag{5}$$

Considering Eq. (4), the following relation is obtained:

$$\kappa_{ij} = \frac{1}{2}e_{jkl}u_{l,ki} \tag{6}$$

where κ can be defined also, the curl of the strain.

2.2. Strain and kinetic energies

The strain energy of the deformed body is assumed to depend on the strain ε and the rotation gradient κ, so that the associated stress quantities are the symmetric Cauchy stress tensor σ and the deviatoric couple stress tensor μ (Mindlin, 1964). It then follows that the strain energy E_s in a deformed isotropic linear elastic material occupying region V is given by

$$E_s = \frac{1}{2}\int_V (\sigma_{ij}\varepsilon_{ij} + \mu_{ij}\kappa_{ij})\, dv \tag{7}$$

In the case of a homogenous continuum composed of unit cells having the form of cubes with characteristics dimension $2a$, the micro-deformation is equal to the gradient of the displacement and the relative deformation is neglected as worked out by Mindlin (1964). Georgiadis and Velgaki (2003) have introduced the following expression for the kinetic energy of a homogeneous continuum with couple stress and micro-inertia effects:

$$E_k = \int_V \left(\frac{1}{2}\rho\dot{u}_i\dot{u}_i + \frac{1}{6}\rho a^2\dot{u}_{j,i}\dot{u}_{j,i}\right) dv \tag{8}$$

In the above equation ρ denotes the mass density of the body. The last term in the above integral indicates the kinetic energy due to the micro-variables.