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Structural modeling, vibration analysis and optimal viscoelastic layer characterization of adaptive sandwich beams with electrorheological fluid core

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ABSTRACT

This paper deals with the vibration analysis of adaptive sandwich beams with electrorheological fluid (ERF) core. In this study, Timoshenko beam theory has been employed to derive the governing equations of motion of variable structure sandwich beams. A consistent procedure is proposed for optimal characterization of the viscoelastic core. Herein, the experimental ASTM E756 method is combined with the computational particle swarm optimization (PSO) to estimate and update the complex shear modulus of the viscoelastic layer. The effects of thickness of the layers and applied electric fields on natural frequencies and modal loss factors are investigated.

1. Introduction

ERFs are produced by a mixture of semiconducting particles within the non-conducting carrier fluids (Powell, 1994). Rheological properties of ERFs are controllable; in fact, these properties change instantaneously by applying electric field. ERFs are used in numerous engineering applications. In particular, the fluids can be incorporated into composite/sandwich structures for vibration control. This is achieved by adjusting the damping and stiffness of the structure by applying the electric field (Coulter et al., 1993; Stanway et al., 1996).

Sandwich beams are used in engineering applications because of high stiffness and strength. Modeling and vibration of viscoelastic sandwich beams has been previously reported in literature; several theoretical and experimental studies have been conducted on the use of ERFs for vibration control of structures with constrained layer damping (CLD) (Allahverdizadeh et al., 2012; Choi et al., 1993). Two well-known models namely Ross, Kerwin and Ungar (RKU), and Mead and Markus are used for modeling the dynamic behavior of viscoelastic material based adaptive structures. A theoretical and experimental investigation for verifying the functionality of these two models in predicting the dynamic behavior of electrorheological (ER) based structures have been done by Don and Coulter (1995). A theoretical model for ER beams with various boundary conditions and semi active vibration control of such adaptive beams were studied by Yalcintas and Coulter (1995a). Vibration properties of a sandwiched beam which is fully or partially treated with an ER fluid layer have been studied experimentally, based on frequency response functions (Haiqing and King, 1997). Dynamic behavior of an adaptive beam with ER elastomer layer as viscoelastic damping material has been considered by Wei et al. (2011); where the properties of elastomer layer are controllable by electric field. Finite element method and experimental analysis were conducted to evaluate the effects of an electric field on the frequency response functions. Critical load and dynamic behavior of a simply supported beam with embedded ER fluid was reported by Yeh and Shih (2005). Parametric instability and dynamic response of an ER beam subjected to a periodic axial force were also investigated.

Mead and Markus (1969, 1970) derived the sixth-order differential equation of motion for transverse vibration of a three-layer sandwich beam assuming shear modulus of the viscoelastic core. Howson and Zare (2005) developed an exact dynamic member stiffness matrix for flexural vibration of three-layered sandwich beams using the closed form solution of the governing differential equation. Kristensen et al. (2008) investigated composite beams with one or more damping layers, and they derived semi-analytical model of the beam using Timoshenko beam theory and finite element method. Banerjee (2001) derived analytical expressions for the frequency equation and mode shapes of composite

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Timoshenko beams with clamp-free boundary condition. Tang and Lumsdaine (2008) examined a partially covered beam with constrained damping layer. The longitudinal normal and shear strain in the viscoelastic layer were considered along with Hamilton's principle to derive equations of motion and boundary conditions. Hu et al. (2005) evaluated several classical models for sandwich beams, and compared the results by the finite-element-based solutions. Rikards et al. used Timoshenko beam theory and finite element method for vibration analysis of laminated viscoelastic composites. The dynamic analysis of sandwich structural elements with viscoelastic middle layer, for different materials and geometric parameters was carried out (Rikards et al., 1993). Lee (1995) studied finite element formulation of a sandwich beam with embedded ERFs. The nonlinear behavior of the ER fluid in quasi-static shear was used in FEM model, and the amplitude-dependent dynamic characteristics of the beam were investigated.

Various investigations have been conducted for characterization of ERFs and viscoelastic materials (Lee and Cheng, 2000). Mahjoob et al. (1995) used modal testing of composite beams for different modes to calculate the dynamic characteristics of viscoelastic layers in the pre-yield regime. The complex modulus of ERF layer was identified based on Mead and Markus (1969, 1970) formulation and experimental results. Using an identification method, viscoelastic characteristics of ERFs have been studied by Esmonde and See. The method is carried out under various applied electric fields for unsteady flow conditions (Esmonde and See, 2009). Choi et al. studied mechanical properties of an ER composite beam in free vibration. Effects of ER fluid and applied electric field on the complex modulus of the beam have been considered. They correlated the derived storage and loss moduli with the rheological properties of the used ER fluid (Choi et al., 1990). Mohammadi et al. investigated rheological models of magnetic field dependent smart materials experimentally. Results of the experimental tests have been used to present the proper models which illustrate the rheological behavior of these fluids (Mohammadi et al., 2010).

In this study, ER fluid sandwiched between two layers, acts as a viscoelastic damping layer with controllable shear modulus in order to attenuate the vibration. Timoshenko beam theory is used along with the Hamilton's principle to derive the governing equations of motion. The presented model of the beam is validated by comparative studies in the literature. A new characterization process that combines ASTM E756 standard and PSO method is proposed to well estimate complex shear modulus of the viscoelastic layer to correlate proposed model with experiment. The effects of electric field and thickness of the layers on the natural frequencies and modal loss factors of the beam are presented. Finally, calculated natural frequencies and modal loss factors based on Timoshenko beam theory are compared with the results based on Euler–Bernoulli beam theory; and the differences of their results are presented.

2. Theoretical formulation

Modeling of an ERF-based sandwich beam and derivation of its governing equations is conducted in this section. ERF-based smart structures are generally used for noise and vibration control. The knowledge of the behavior of ERFs within the small strain range becomes more important in the study of the structures. Generally, ERFs are assumed to be linearly viscoelastic materials, in which the complex shear modulus depends on applied electric field. With this assumption, modeling of the ERF-based adaptive beam has been accomplished by using a modified sandwich beam theory (Lee, 1995). In addition to employ the elastic governing equation, all layers of the structure are assumed to be elastic. Then, the elastic constants in the derived equation of the viscoelastic layer is simply replaced by its corresponding property in viscoelastic material that is the complex modules (Daya et al., 2004).

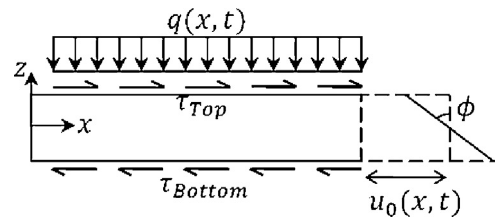

Fig. 1. Displacement field of the single-layer beam.

Governing equations of motion for dynamic behavior of sandwich beams with an electro-rheological core were previously derived with different assumptions such as ignoring shear deformation of faces (applying the Euler–Bernoulli beam theory). In this study, Timoshenko beam theory is employed to include shear effects of beam and constraining layers. The considered assumptions to derive equations of motion are: (a) Kinetic energies of longitudinal and rotational motions are much smaller than that of transverse motion. (b) External forces are applied only in transverse direction; there is no axial force (c) The electro-rheological core is only under shear deformation and Young's modulus of the ERF core is negligible in comparison with the constraining elastic layers; therefore, the strain energy of normal stresses through the ER layer is neglected. (d) All layers have the same transverse displacement (w), (e) There is no slippage between adjacent layers.

2.1. Modeling of constraining layers

In order to employ Timoshenko beam theory for the faces, it is necessary to derive the equations of motions for a single-layer beam, under transverse loading and shear stress acting on its top and bottom faces as shown in Fig. 1. These equations can then be applied on each layer of the sandwich beam. The variational method is employed to obtain the governing equations using D'alembert principle and the theorem of extremum potential energy. According to Fig. 1, the displacement field of the single-layer beam can be presented as:

$$u(x,z,t) = u_0(x,t) - z\phi(x,t) \tag{1}$$

$$v(x,z,t) = 0 \tag{2}$$

$$w(x,z,t) = w(x,t) \tag{3}$$

Then, the non-zero strains, based on the Timoshenko beam theory, are

$$\varepsilon(x,z,t) = u_0(x,t)_{,x} - z\phi(x,t)_{,x} \tag{4}$$

$$\gamma_{xz}(x,z,t) = w(x,t)_{,x} - \phi(x,t) \tag{5}$$

Since the actual shear stress through the cross section is not constant, a correction factor, κ, is introduced such that

$$\tau_{xz}(x,z,t) = \kappa G\{w(x,t)_{,x} - \phi(x,t)\} \tag{6}$$

where κ is shear coefficient. For a solid rectangular cross section, shear coefficient has been reported as (Gere and Timoshenko, 1976):

$$\kappa = \frac{10(1+\nu)}{12+11\nu} \tag{7}$$

where ν is Poisson's ratio. The variation in the internal energy, external force work and the potential energy can be expressed as:

$$\delta U = \iint (\sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz})dAdx \tag{8}$$