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Anisotropic Mullins stress softening of a deformed silicone holey plate

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ABSTRACT

Rubber like materials parts are designed using finite element code in which more and more precise and robust constitutive equations are implemented. In general, constitutive equations developed in literature to represent the anisotropy induced by the Mullins effect present analytical forms that are not adapted to finite element implementation. The present paper deals with the development of a constitutive equation that represents the anisotropy of the Mullins effect using only strain invariants. The efficiency of the modeling is first compared to classical homogeneous experimental tests on a filled silicone rubber. Second, the model is tested on a complex structure. In this aim, a silicone holey plate is molded and tested in tension, its local strain fields are evaluated by means of digital image correlation. The experimental results are compared to the simulations from the constitutive equation implemented in a finite element code. Global measurements (i.e. force and displacement) and local strain fields are successfully compared to experimental measurements to validate the model.

1. Introduction

Natural and synthetic elastomers are widely used in industrial design and numerical simulations are often used to develop new parts. These simulations are fundamental in the optimization process of structure design. One of the main difficulties of the engineers is the choice of an adapted constitutive equation able to represent the behavior of the rubber like materials. The choice is often oriented toward a hyperelastic constitutive equation. Finite element codes present a large choice of strain energy densities, even if the Mooney (1940) constitutive equation stays one of the most used.

However rubber like materials present a lot of other phenomena than pure hyperelasticity to take into account, as for example: the Mullins effect, the hysteresis and the time dependence. The Mullins effect is very important as the mechanical behavior of the material can totally change after a first loading as it depends both on the maximal deformation and the loading direction. For instance, for the study of cracks, the local deformation (i.e. near the tip) are much superior to the global deformation initially inflected to the structure, so a softer behavior is not adapted. Nevertheless the Mullins effect could be ignored for particular studies, for instance Lion (1997) analyzed the hysteresis of elastomers for second loadings. Thus for some rubber parts, it is very important to take into account the Mullins effect, but very few constitutive equations are implemented into industrial finite element codes. Eventhough, many researchers have developed isotropic constitutive equations for the Mullins effect and proposed a finite element implementation (see for example Miehe, 1995; Miehe and Keck, 2000; Kaliske et al., 2001; Chagnon et al., 2006; Guo et al., 2006; Cantournet et al., 2009; Gracia et al., 2009).

Another important point is that, many new experimental data are proposed in the literature to emphasize that the Mullins effect is strongly anisotropic (Muhr et al., 1999; Park and Hamed, 2000; Pawelski, 2001; Besdo and Ihlemann, 2003; Laraba-Abbes et al., 2003; Diani et al., 2006a; Hanson et al., 2005; Itskov et al., 2006; Machado et al., 2012b; Dorfmann and Pancheri, 2012). Different constitutive equations have been proposed but they are not adapted to finite element implementation. The only formulation implemented in a finite element code was proposed by Göktepe and Miehe (2005) who used the approach of Miehe et al. (2004) introducing a directional damage depending on the energy in the considered direction. The use of the energy as a governing parameter needs an optimization loop whereas a strain formulation avoids this loop. In this way, the idea is here to propose a new formulation that only depends on the strain state, avoiding the calculation of energies which permits to have an explicit expression of the stress in function of the strain. Some constitutive equations have been developed for living tissues (Peña et al., 2009; Bose and Dorfmann, 2009; Kroon and Holzapfel, 2008) but they are limited to materials presenting two reinforced directions. In this way, in this paper, a new anisotropic model based on strain invariant formulation, is proposed and implemented in a finite element code.

Recently, Machado et al. (2010, 2012b) developed a large database for a filled silicone rubber including on one hand cyclic classical experimental tests and on the other hand uniaxial tests

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realized after different uniaxial and biaxial tension tests. This database is, here, used to build a new constitutive equation easily implementable in finite element codes. In Section 2, the constitutive equation is detailed and the ability of the model to describe recent experimental data is discussed. In Section 3, the subject of the numerical implementation of the model is treated. In Section 4, the ability of the model to describe complex structures is tackled by means of a specific test on a rectangular plate containing five holes. The global and local estimations of the model are compared to experimental measures. Finally, Section 5 contains some concluding remarks of the modeling.

2. Anisotropic modeling of the Mullins effect

2.1. General formulation in strain invariants

Different anisotropic approaches to model Mullins effect were proposed in literature, but none of them was only expressed in term of strain invariants. Shariff (2006) and Itskov et al. (2010) took into account three principal damage directions to reproduce a special behavior in the direction orthogonal to loading. In a more general way, the spatial repartition of Bazant and Oh (1986) was used by many authors to create an anisotropic model. Diani et al. (2006b) and Dargazany and Itskov (2009) generalized the network evolution proposed by Marckmann et al. (2002) to an anisotropic approach by taking into account the maximum elongation in each spatial direction. Later Merckel et al. (2011, 2012) introduced a new framework and proposed a softening anisotropic criterion adapted to complex loading states.

The stress softening phenomenon has often been associated to the presence of fillers in the rubber, but Harwood et al. (1965) showed that stress softening can also occur in unfilled rubbers, even if it is reduced compared to filled rubbers. For silicone rubbers, Meunier et al. (2008) observed no Mullins effect for an unfilled one, whereas Machado et al. (2010) observed stress softening for a filled one. As a consequence, it can be considered that fillers in silicone rubbers are mainly responsible of the Mullins effect. Thus, as proposed by Govindjee and Simo (1992) the strain energy density $\mathcal{W}$ is additively decomposed into two parts: one that represents the energy density of the chains linked to other chains $\mathcal{W}_{cc}$ and an other part that represents the energy density of the chains linked to filler $\mathcal{W}_{cf}$. The total strain energy density is $\mathcal{W} = \mathcal{W}_{cc} + \mathcal{W}_{cf}$. It is considered that only $\mathcal{W}_{cf}$ can evolve with the Mullins effect as in Göktepe and Miehe (2005). As a consequence $\mathcal{W}_{cc}$ is represented by a classical hyperelastic isotropic energy density and $\mathcal{W}_{cf}$ must be represented by an anisotropic strain energy that can evolve with the deformation history of the material. The ideal representation would be to propose a full integration of all spatial directions as proposed by Wu and Giessen (1993) in hyperleasticity, but it is not adapted to finite element implementation. A spatial discretization is needed. Forty-two initial spatial directions, noted $\mathbf{A}^{(i)}$, are introduced, these directions are those proposed by Bazant and Oh (1986). Therefore, the strain energy density is written as:

$$\mathcal{W} = \mathcal{W}_{cc}(I_1, I_2) + \sum_{i=1}^{n} \omega^{(i)} \mathcal{F}^{(i)} \mathcal{W}_{cf}^{(i)}(I_4^{(i)}) \quad (1)$$

where I_1, I_2 are the first and second strain invariants of the right Cauchy–Green strain tensor $\mathbf{C}$. The strain in each direction (i) is defined by means of $I_4^{(i)} = \mathbf{A}^{(i)}.\mathbf{C}\mathbf{A}^{(i)}$. $\omega^{(i)}$ represents the weight of each direction and $\mathcal{F}^{(i)}$ is the Mullins effect evolution function. The initial directions $\mathbf{A}^{(i)}$ are transformed in $\mathbf{a}^{(i)}$ by $\mathbf{a}^{(i)} = \mathbf{F}\mathbf{A}^{(i)}$, where $\mathbf{F}$ is the deformation gradient.

Classically, in an isotropic approach, the evolution function $\mathcal{F}^{(i)}$ would be written through the strain energy density, but Chagnon et al. (2004) showed that the first invariant can also be used. In an anisotropic approach, the elongation in each direction is used Diani et al. (2006b), knowing that the elongation is the square root of the invariant I_4. According to the conclusions of Machado et al. (2012b), it is chosen to describe the stress-softening function according to I_1 and $I_4{}^{(i)}$. For each direction (i), an evolution function which depends on three terms $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_3$ is proposed:

$$\mathcal{F} = 1 - \mathcal{F}_1({I_1}^{\max} - I_1)\mathcal{F}_2({I_4}^{\max(i)} - {I_4}^{(i)})\mathcal{F}_3\left(\frac{{I_4}^{\max(i)}}{{I_4}^{\max}}\right) \quad (2)$$

where ${I_1}^{\max}$ and ${I_4}^{\max(i)}$ represent the maximum values taken during the material history by I_1 and ${I_4}^{(i)}$ respectively. ${I_4}^{\max} = \max_i({I_4}^{(i)})$ is the maximum dilatation in space and time. As proposed by Zuñiga and Beatty (2002), a function that is constant during first loading and that evolves with the maximum and current deformations is imposed for the evolution function.

2.2. A particular form for evolution function

A Mooney (1940) constitutive equation is chosen for $\mathcal{W}_{cc}$, and a Kaliske (2000) quadratic equation $K(I_4^{(i)} - 1)^2$ is chosen for $\mathcal{W}_{cf}^{(i)}$, where $K^{(i)}$ is a material parameter. A first particular form is proposed for the stress-softening function, considering that a minimum of parameters should be introduced:

$$\mathcal{F}^{(i)} = 1 - \eta\sqrt{\frac{I_{1\max} - I_1}{I_{1\max} - 3}}\left(\frac{I_{4\max}^{(i)} - I_4^{(i)}}{I_{4\max}^{(i)} - 1}\right)\left(\frac{I_{4\max}^{(i)}}{I_{4\max}}\right)^4 \quad (3)$$

In this way, the evolution function depends only on one parameter: η which controls the stress softening for a given direction, this parameter is without unity. The large experimental database proposed by Machado et al. (2010, 2012b) on a filled silicone rubber is used to fit the model. These experimental results are decomposed into three parts: first the classical uniaxial tension, planar tension and biaxial tests realized by means of a bulge test (Machado et al., 2012a), second the complex tensile tests with change of directions after the first loading; and third biaxial tensile tests followed by uniaxial tensile tests. The three hyperelastic parameters C_1, C_2 and $K^{(i)}$ are obtained by fitting the different first loading curves. All the $K^{(i)}$ parameters are chosen equal as the material is initially isotropic. The followings values are obtained: $C_1 = 0.05$ MPa, $C_2 = 0.03$ MPa and $\forall i$ $K^{(i)} = 0.10$ MPa. The last parameter that characterizes the stress softening is obtained by minimizing the errors on the second loading curves for all the tests. The value $\eta = 4$ is obtained

The simulations of the cyclic uniaxial tensile, pure shear and equibiaxial tensile tests are presented in Fig. 1. It appears that the model describes well the stress softening for all these tests. The model predicts well uniaxial and pure shear tests whereas equibiaxial first loading curve is underestimated. This phenomenon is expected since first loading depends only on the hyperelastic equation. As explained by Marckmann and Verron (2006), it is difficult to fit all the different tests with the same energy density.

The proposed model is now compared with the experimental data of the two complex pre-conditioning methods. First, Fig. 2, presents the results for tensile tests with a change of loading direction between the first and second loadings. The results from the model do not superimpose exactly experimental data, but all trends are quite well described for the different directions. Second, the model predictions are compared with biaxial pre-stretching tests results. The biaxial loading is characterized by the biaxiality ratio defined as $\mu = \ln(\lambda_{\min})/\ln(\lambda_{\max})$ (where $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum in-plane principal elongations). Tests with different biaxiality ratios were used for the simulation. The comparison of the second loading curves is presented in Fig. 3. It appears