Mechanics Research Communications 38 (2011) 602–606

Contents lists available at SciVerse ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

A two scale model of porous rocks with Drucker–Prager matrix: Application to a sandstone

J. Lin[a], J.-F. Shao[b], D. Kondo[c,*]

[a] Institut de Mécanique des Fluides et des Solides, Université de Strasbourg, 67000 Strasbourg, France
[b] Laboratoire de Mécanique de Lille, UMR 8107 CNRS, USTL, 59655 Villeneuve d'Ascq, France
[c] Institut D'Alembert, UMR 7190 CNRS, Université Pierre Et Marie Curie (UPMC), 75005 Paris Cedex, France

ARTICLE INFO

Article history:
Received 12 June 2010
Received in revised form 16 July 2011
Available online 16 September 2011

Keywords:
Drucker–Prager matrix
Ductile behavior
Micromechanics
Gurson-type model
Porous rocks
Triaxial compression tests

ABSTRACT

The present paper is devoted to a micromechanical model of porous rocks and its application to a sandstone. This original model takes advantage of a recent homogenization-based macroscopic yield function which couples Drucker–Prager type plasticity of the solid matrix and evolving porosity. Its formulation and implementation are described. Application to a Vosges sandstone shows that, except for very low confining pressures for which the mechanical behavior is quasi-brittle, the model predicts well the ductile behavior at moderate or high confining pressures (for which the pore collapse mechanism is expected to play a dominant role).

1. Introduction

It is commonly recognized that evolving porosity strongly affects the macroscopic mechanical behavior of cohesive geomaterials undergoing plastic deformation (Li et al., 2009; Menéndez et al., 1996; Zimmermann, 1991). Since the pioneering work of Gurson (1977), a relevant way to incorporate the effects of voids in ductile materials behavior consists in deriving the macroscopic yield function from a limit analysis approach. The Gurson yield function predicts the effects of spherical voids on plastic behavior of metallic porous materials whose matrix obeys to a von Mises criterion. It has been widely used to formulate macroscopic constitutive models allowing to investigate appropriately voids growth process (Tvergaard and Needleman, 1984; Besson, 2001). Despite the interest of these studies, the Gurson model fails to be applicable to non metallic materials such as polymers or geomaterials for which the solid matrix generally exhibits a pressure-sensitivity. For this class of materials, Jeong (2002), and more recently Guo et al. (2008) extend the Gurson limit analysis-based criterion by considering ductile porous media having a Drucker–Prager type matrix.

The main objectives of the present study are: (i) to formulate a micromechanical constitutive model of ductile porous materials having a Drucker–Prager (pressure-sensitive) matrix; (ii) to implement this model and assess its capabilities by comparing the predictions to experimental data on a porous sandstone.

2. Macroscopic criterion of porous media with plastically compressible matrix

In order to describe the mechanical behavior of geomaterials, it is desirable to develop a constitutive model of ductile porous material taking into account the plastic compressibility of the matrix. This section is devoted to the presentation of a new Gurson-type model based on a recent macroscopic criterion, derived by Guo et al. (2008) from the limit analysis of a hollow sphere whose solid matrix obeys to a Drucker–Prager criterion.

2.1. Methodology of derivation of Gurson-type model

Let us consider a representative elementary volume (*r.e.v.*) Ω of a porous material with porosity f. The derivation of the Gurson-type model presented below is based on the rigorous framework of Limit Analysis which can be found in de Buhan (1986) and Suquet (1985). The textbooks Leblond (2003) and Dormieux et al. (2006) also introduced the main concepts of this theory for the derivation of the macroscopic strength of ductile porous media (see also Dormieux and Kondo, 2010). Let $\mathbf{\Sigma}$ and $\mathbf{D}$, respectively, denote the macroscopic stress and strain rate tensors. $\mathcal{V}(\mathbf{D})$ is the set of microscopic

* Corresponding author. Tel.: +33 1 44 27 54 85; fax: +33 1 44 27 52 59.
E-mail address: djimedo.kondo@upmc.fr (D. Kondo).

0093-6413/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mechrescom.2011.08.005

velocity fields, $\underline{v}(\underline{z})$, kinematically admissible with $\mathbf{D}$. These velocity fields comply with uniform strain rate boundary conditions:

$$\mathcal{V}(\mathbf{D}) = \{\underline{v},\ \underline{v}(\underline{z}) = \mathbf{D}\cdot\underline{z} \quad (\forall \underline{z} \in \partial\Omega)\} \tag{1}$$

Let us consider a microscopic stress field $\boldsymbol{\sigma}(\underline{z})$ in equilibrium and related to the macroscopic stress tensor $\boldsymbol{\Sigma}$, in the sense of the average rule $\boldsymbol{\Sigma} = 1/|\Omega| \int_\Omega \boldsymbol{\sigma}\, dV$. Hill's lemma states that:

$$\boldsymbol{\Sigma} : \mathbf{D} = \frac{1}{|\Omega|}\int_\Omega \boldsymbol{\sigma} : \mathbf{d}\, dV \tag{2}$$

with $\mathbf{d} = (1/2)(\text{grad}\,\underline{v} + {}^t\text{grad}\,\underline{v})$, the microscopic strain rate tensor. The strength of the solid phase is characterized by the convex set G^s of admissible stress states, which in turn is defined by a convex strength criterion $\phi^s(\boldsymbol{\sigma})$:

$$G^s = \{\boldsymbol{\sigma}, \phi^s(\boldsymbol{\sigma}) \leq 0\} \tag{3}$$

The dual definition of the strength criterion consists in introducing the support function $\pi^s(\mathbf{d})$ of G^s, which is defined on the set of symmetric second order tensors $\mathbf{d}$ and is convex with regard to $\mathbf{d}$:

$$\pi^s(\mathbf{d}) = \sup(\boldsymbol{\sigma} : \mathbf{d}, \boldsymbol{\sigma} \in G^s) \tag{4}$$

$\pi^s(\mathbf{d})$ represents the microscopic maximum "plastic" dissipation. Its macroscopic counterpart is defined as:

$$\Pi^{hom}(\mathbf{D}) = (1-f)\inf_{\underline{v}\in\mathcal{V}(\mathbf{D})}\left[\frac{1}{|\Omega^s|}\int_{\Omega^s}\pi^s(\mathbf{d})\,dV\right] \tag{5}$$

Using Eq. (2) together with (5), it can be shown that Π^{hom} is the support function of the domain G^{hom} of macroscopic admissible stresses:

$$\Pi^{hom}(\mathbf{D}) = \sup(\boldsymbol{\Sigma} : \mathbf{D}, \boldsymbol{\Sigma} \in G^{hom}) \tag{6}$$

The limit stress states at the macroscopic scale are shown to be of the form:

$$\boldsymbol{\Sigma} = \frac{\partial \Pi^{hom}}{\partial \mathbf{D}} \tag{7}$$

The above approach has been implemented by Gurson (1977), assuming a Von Mises criterion for the solid matrix and considering the following simplifications.

- The first simplification consists in representing the morphology of the porous material by a hollow sphere instead of the *r.e.v.* Let R_e (resp. R_i) denote the external (resp. cavity) radius. The volume fraction of the cavity in the sphere is equal to the porosity $f = (R_i/R_e)^3$.
- Instead of seeking the infimum in Eq. (5), $\Pi^{hom}(\mathbf{D})$ is estimated by considering a particular microscopic velocity field $\underline{v}(\underline{z})$ composed of an homogeneous part and a radial heterogeneous one (corresponding to the solution of the hollow sphere obeying to a Von Mises criterion and subjected to an external hydrostatic pressure).

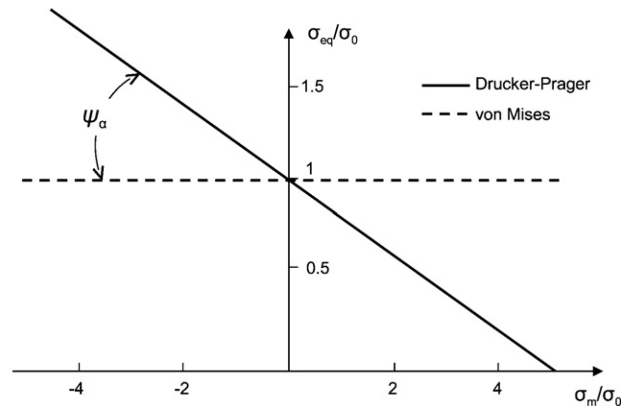

Fig. 1. A sketch of the Drucker–Prager criterion and the von Mises criterion.

2.2. *A brief summary of the criterion of Guo et al. (2008) in the case of a Drucker–Prager matrix*

The aim of this subsection is to briefly summarize the methodology leading to the macroscopic yield function (of the porous medium) established by Guo et al. (2008) who follow the Gurson approach, applying it to the case where the solid phase is plastically compressible and obeys to a Drucker–Prager criterion. The later is defined as:

$$\phi^s(\boldsymbol{\sigma}) = \sigma_{eq} + 3\alpha\sigma_m - \sigma_0 \tag{8}$$

where σ_m is the hydrostatic part of the local stress tensor $\boldsymbol{\sigma}$, $\sigma_{eq} = \sqrt{(3/2)\boldsymbol{\sigma}' : \boldsymbol{\sigma}'}$ (with $\boldsymbol{\sigma}'$ the deviatoric part of $\boldsymbol{\sigma}$) is the local von Mises equivalent stress. σ_0 is linked to the yield stress of the matrix under pure shear ($\sigma_m = 0$). α is related to the friction angle of the matrix ψ_α by $\tan\psi_\alpha = 3\alpha$. Note that the Von Mises criterion corresponds to the particular case of $\alpha = 0$. As shown in Fig. 1, ψ_α represents the slope of the criterion in $\sigma_{eq} - \sigma_m$ stress space.

The corresponding support function $\pi^s(\mathbf{d})$ (local plastic dissipation) then reads:

$$\pi^s(\mathbf{d}) = \sigma_0 d_{eq} \tag{9}$$

in which, due to the plastic compressibility of the solid matrix, d_{eq} is related to the volumetric strain tr$\mathbf{d}$ by:

$$\text{tr}\mathbf{d} = 3\alpha d_{eq} \quad \text{with}\ d_{eq} = \sqrt{\frac{2}{3}\mathbf{d}' : \mathbf{d}'} \tag{10}$$

where $\mathbf{d}'$ represents the deviatoric part of $\mathbf{d}$. For the determination of the macroscopic yield function, Guo et al. (2008) considered a velocity field consisting, as in the Gurson work, in a radial heterogeneous part (associated here to a Drucker–Prager matrix) and an homogeneous one. In the cylindrical frame (coordinates r, θ, z), this field reads:

$$\underline{v}(\underline{z}) = C_0\left(\frac{b}{r}\right)^{3/s}(\rho\underline{e}_\rho + z\underline{e}_z) + C_1\rho\underline{e}_\rho + C_2 z\underline{e}_z \tag{11}$$

where C_0, C_1 and C_2 are three constants to be determined, $s = 1 \pm 2\alpha$ when $C_0 \gtrless 0$ and ρ is such that $r = \sqrt{\rho^2 + z^2}$. It has been shown that $\Pi^{hom}(\mathbf{D})$ (Eq. (5)), computed with (9) in which d_{eq} is obtained by considering the velocity field (11), reads (Guo et al., 2008):

$$\Pi^{hom}(\mathbf{D}) = \frac{|C_1 - C_2|}{3}\int_f^1\int_0^\pi \sqrt{\left[1 + \frac{1}{2}\left(3\cos^2\theta - 1\right)\zeta\right](1 + \omega^2 x^{-2/s})}\sin\theta\, d\theta\, dx \tag{12}$$

where

$$\zeta = \frac{2\omega x^{-1/s}}{1 + \omega^2 x^{-2/s}}\text{sign}(C_1 - C_2) \quad \text{and} \quad \omega = \frac{3C_0}{s|C_1 - C_2|}.$$