Mathematical theory of evolution of loaded solids and media

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We are lucky to live in a complex and astonishing nonlinear world.
H. Knyazeva, S. Kurdyumov

The purpose of computing is insight, not numbers.
R. Hamming

The paper puts forward an evolutionary approach to describing the deformation response of loaded solids and media, which is based on the concepts of nonlinear dynamics. The deformation response is considered to mean destruction processes in solid media in external force fields, i.e., the processes of inelastic deformation and simultaneous failure. The mathematical theory of evolution of solids and media is shown to rest on the equations of solid mechanics as fundamental equations of mathematical physics describing the most general laws of conservation of mass, momentum, angular momentum and energy. The whole variety of physical mechanisms of inelastic (plastic) deformation and dilatation, i.e., the generation of discontinuities of different scales and physical nature (vacancies, pores, micro- and mesodamage, etc.) is integrally described through assigning nonlinear response functions of the loaded medium by evolutionary constitutive equations of the first and second group. These equations are thus derived on the basis of the leading physical mechanisms of the studied scale.

It is shown that by varying only the correlation between positive and negative feedbacks (other things equal) the response of the loaded medium varies from the typical plastic flow to brittle failure. The procedure of introducing the real process time in the model is proposed. It allows solving problems on shock wave loading as well as problems of geodynamics and plate tectonics with characteristic times of millions of years.

Keywords: evolution, nonlinear dynamics, mathematical theory, a deformed solid, plasticity, failure

1. Introduction. Synergetics and real natural and physical nonlinear systems

Historians of the science note that at different stages of accumulation of the scientific knowledge leading disciplines change (vapor century, electricity century, atom century, electronics century…), whereas certain ideas remain permanent for a long time. A common subject for many disciplines is the evolution from simple to complex forms.

Different tasks have been posed at different stages of the subject study. Presently the general theory of evolution aims at developing the mathematical theory of self-organization in different systems. Among studied nonlinear systems an important place is occupied by solids, namely, materials and strength media. Their evolution and primarily the evolution of their strength properties up to the failure point under various external actions are of practical interest. We wish to know the evolution of the stress-strain state at each point of loaded solids and media, to describe self-organization in a medium and degradation of strength characteristics of materials and media, and to predict failure time and site.

Thus, the task of mathematical simulation of the medium response to loading is changed into one of the medium evolution in acting force fields.

For continuum mechanics, and for the mechanics of deformed solids and media in particular, the problem of evolution can be stated as that of the formation of different structures in a loaded structureless continuum, of their change, conditions for their formation, decay and transformation from one form to another.

Such problem statement inevitably requires an adequate mathematical theory describing self-organization in solids and media.

The article title "Mathematical theory of evolution…" should be given certain justification, if not vindication. Modern nonlinear dynamics or synergetics in its broad sense pretends in some degree to this role (i.e., the role of the

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mathematical theory of evolution). However, its application to real physical objects and to the solution of applied problems meets great difficulties. Practical tasks require investigators to predict the evolution of real physical nonlinear systems not in general but with the maximum possible thorough description of details of studied processes.

This brings up some complex questions. What do synergetics and studied general properties of the fundamental equations of nonlinear dynamics provide for understanding the evolution of real dynamic systems, specifically the evolution of the stress-strain state in deformed solids? What equation should be used to simulate the evolution of loaded solids and media?

Tasks related to self-organization is the main subject of synergetics or nonlinear dynamics. Self-organization laws and general evolutionary scenarios for very different systems, which are disclosed by nonlinear dynamics, turn the latter into an interdisciplinary science. The paradoxicality of synergetics and its conclusions as well as its generality (interdisciplinarity) distinguish it fundamentally from other conventional sciences. However, the authors [1] reason that it provides no simple and clear recipes what and how one should calculate. Rather it helps us to correctly ask questions and formulate problems. In their opinion, one cannot expect concrete results from a theory of everything [1].

After determining that a specific system behaves by synergetic laws, the researcher inevitably meets a nontrivial problem of developing an adequate mathematical model in a language of nonlinear dynamics, i.e., an evolutionary problem. In an overwhelming majority of cases, with rare lucky exceptions (detailed below), such work is related to expanding of nonlinear dynamics scope and introduction of new models of equations and new understanding.

The evolution of different systems is peculiar in that the course of events in them can vary and a smooth course of events can pass to blow-up modes of their development for very short times. At these modes a system changes cardinally and acquires new structures and properties. Systems become self-organized due to a transition through dynamic chaos and decay of old structures to the formation of new structures.

All these peculiarities of the system evolution are studied rather thoroughly due to investigating special features of solutions of fundamental synergetic equations [1–10] and are widely discussed in the extensive literature on the subject [1–15].

The study of general properties of fundamental synergetic equations (among which are nonlinear equations of Ginsburg–Landau, Schrödinger, thermal conductivity, Fokker–Planck, Korteweg–de Vries, etc. as well as equations derived from more general systems, for example, from partial differential equations, and representing simpler systems of ordinary nonlinear differential equations) provides general ideas of evolution of open dynamic nonlinear systems. We have studied different evolutionary scenarios, stability loss conditions, scenarios of a transition to chaos, peculiarities of diffusion chaos, generation of different nonperiodic turbulent modes, and many others. The synergetics achievements are unquestionable and cannot be overestimated. This new scientific paradigm results in a new worldview and new understanding of evolution processes of open systems. However, various phenomena being qualitatively similar show radical differences in details. The study of evolution of real dynamic systems and solution of specific applied tasks prove to be almost unsolvable problems. Any solution of a specific applied task requires that both evolutionary laws and character be understood and all stages of a studied process be detailed. The current situation is clear. Real physical processes and applied tasks are described by systems of partial differential equations, for which the analysis of general properties of their solutions is still an unsolvable problem. Simplification of equations allows analyzing special cases being often far from the reality.

By Galerkin's method from hydrodynamic equations for convective flow in 1963 Lorenz obtained a system of equations (three ordinary differential equations) named after him. He was the first who obtained and studied a strange attractor, i.e., discovered deterministic chaos. However, analyzing very interesting and thorough Lorenz's model we learn nothing about the initial flow described by hydrodynamic equations! To find concrete peculiarities of flow and to solve applied problems one should solve the total initial system of partial differential equations, which also contains modes discovered at studying Lorenz's system of equations. However, now we know what should be searched and under what conditions.

The well-studied basic models of synergetics directly used as mathematical models for a certain studied process are rarely effective and often naive. For example, a nonlinear equation of thermal conductivity (or another nonlinear equation) is used as a fundamental equation to study plastic deformation development. Temperature is replaced by plastic deformation and coefficients are chosen from experiments. However, such "simulation" gives no consideration to the stress-strain state, shear-induced stress relaxation, plastic flow, etc. In fact, a nonlinear process completely unrelated to deformation is studied. Numerous similar examples can be given. Certain speculations on fashionable synergetics appear in many papers. Simple variants of "evolutionary" equations of type (1) are written and posed as a medium "model".

Prigogine proposes to describe the evolution of dissipative systems by the following system of equations [6, 7]

$$\frac{\partial X_i}{\partial t} = F_i(X_1, ..., X_n, x^j, t), \quad (1)$$

where $X_i(x^j, t)$ is the total set of macroscopic variables of a nonlinear system as functions of spatial coordinates x^j and time t and F_i is the functions depending on both