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Buckling analysis of FGM circular shells under combined loads and resting on the Pasternak type elastic foundation

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ABSTRACT

In this study, the buckling analysis of functionally graded material (FGM) circular truncated conical and cylindrical shells subjected to combined axial extension loads and hydrostatic pressure and resting on a Pasternak type elastic foundation is investigated. The critical combined loads of FGM truncated conical shells with or without elastic foundations have been found analytically. The appropriate formulas for FGM cylindrical shells with and without elastic foundations are found as a special case. Several examples are presented to show the accuracy and efficiency of the formulation.

1. Introduction

One of the interesting problems in engineering is the static and dynamic analysis of beams, plates and shells resting on elastic foundations. There are different approaches to analyze the interaction between a structure and an ambient medium. Most earthen soils can appropriately be represented by a mathematical model from Pasternak, whereas sandy soils and liquids can be represented by Winkler's model (Pasternak, 1954; Kerr, 1964). The bending, vibration, buckling and postbuckling studies for beams and plates on elastic foundations subjected to various loads are available in the literature. To a far lesser extent than beams and plates, the static and dynamic analyses of shells on elastic foundations have also been studied. Most of the research has been limited to the stability and vibration analyses of cylindrical shells on elastic foundations (Sun and Huang, 1988; Paliwal and Pandey, 1998; Ng and Lam, 1999; Civalek, 2005; Shen, 2009).

Functionally graded materials (FGMs) are a new class of advanced composites characterized by the gradual variation in composition, microstructure and material properties. These materials have emerged from the need to enhance material performance (Koizumi, 1993). The mechanical behavior of FGMs shells, such as bending due to mechanical loads, free vibration, stability and buckling, etc., have also been studied by many scientists (Reddy and Chin, 1998; Pitakthapanaphong and Busso, 2002; Batra, 2006; Viola and Tornabene, 2009; Matsunaga, 2009; Sofiyev, 2010). Another important issue belong to the mechanical behavior is the interaction of FGMs and elastic foundations. Bending, stability and vibration analyses of FGMs are quite limited, especially of those on elastic foundations (Huang et al., 2008; Sheng and Wang, 2008; Malekzadeh, 2009; Pradhan and Murmu, 2009).

The bucking problem of FGM shells under combined loads and resting on a Pasternak type elastic foundation has not been studied yet. In the present work, an attempt is made to address this problem.

2. Theoretical formulation

Consider a circular truncated conical shell as shown in Fig. 1, which h is the thickness of the conical shell, R_1 and R_2 indicate the radii of the cone at its lower and upper ends, respectively. L is the length and γ is the semi-vertex angle of the conical shell. S_1 and S_2 are the distances from the vertex to the lower and upper bases, respectively. A set of curvilinear coordinates (z, θ, S) is located on the middle surface. Furthermore, the z-axis is always normal to the moving S-axis, lies in the plane generated by the S-axis and the axis of the cone, and points inwards. The θ-axis is in the direction perpendicular to the S–z plane.

The FGM truncated conical shell is subjected to combined axial extension loads and hydrostatic pressure. Two types of combined loads are given below (see Fig. 2a and b):

(a) The FGM truncated conical shell is subjected to combined hydrostatic pressure and an axial tension load that applied to

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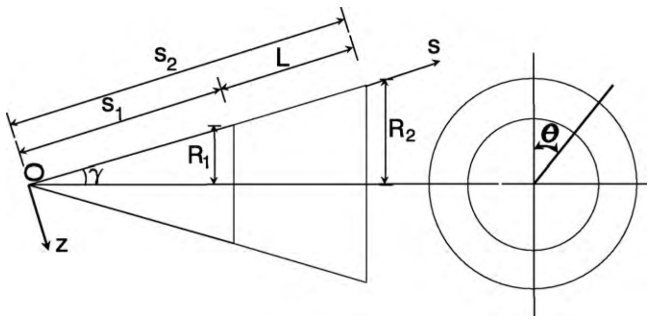

Fig. 1. Geometry of truncated conical shell.

the lower base (see Fig. 2a):

$$T_1^0 = \frac{0.5P_{ax}S_2^2 \tan\gamma}{S} - 0.5P_H S \tan\gamma;$$
$$T_2^0 = -P_H S \tan\gamma; \quad T_{12}^0 = 0 \tag{1}$$

where T_1^0 and T_2^0 are the membrane forces in the curvilinear coordinate directions S and θ, respectively and T_{12}^0 is the membrane shear force for the condition with zero initial moments. P_H is a hydrostatic pressure and P_{ax} is an axial tension load that applied to the lower base of the cone.

(b) The FGM truncated conical shell is subjected to combined hydrostatic pressure and an axial tension load that applied to the upper base (see Fig. 2b):

$$T_1^0 = \frac{0.5P_{ax}S_1^2 \tan\gamma}{S} - 0.5P_H S \tan\gamma;$$
$$T_2^0 = -P_H S \tan\gamma; \quad T_{12}^0 = 0 \tag{2}$$

Types of loads (1) and (2), which are listed above, can be expressed in a unique form:

$$T_1^0 = \frac{0.5P_{ax}Q^2 \tan\gamma}{S} - 0.5P_H S \tan\gamma;$$
$$T_2^0 = -P_H S \tan\gamma; \quad T_{12}^0 = 0 \tag{3}$$

where Q is a symbol and taking the following two different meanings:

(a) When $Q = S_1$, the axial tension load is applied to the lower base of the truncated cone.
(b) When $Q = S_2$, the axial tension load is applied to the upper base of the truncated cone.

The FGM truncated conical shell is resting on an elastic foundation. For the elastic foundation, it is assumed the two-parameter elastic foundation model proposed by Pasternak (1954). The foundation medium is assumed to be linear, homogenous and isotropic. The bonding between the truncated conical shell and the foundation is perfect and frictionless. The foundation interface pressure N may be expressed as

$$N = K_w w - K_p\left(\frac{\partial^2 w}{\partial S^2} + \frac{1}{S}\frac{\partial w}{\partial S} + \frac{1}{S^2}\frac{\partial^2 w}{\partial \varphi^2}\right) \tag{4}$$

where K_w (N/m^3) is the modulus of subgrade reaction for the foundation, K_p (N/m) is the shear modulus of the subgrade, $\varphi = \theta \sin\gamma$, w is the displacement of the middle surface in the normal direction, positive towards the axis of the cone and assumed to be much smaller than the thickness. Note that by setting $K_p = 0$, the Pasternak model becomes that of the Winkler foundation model (Pasternak, 1954; Kerr, 1964; Sun and Huang, 1988).

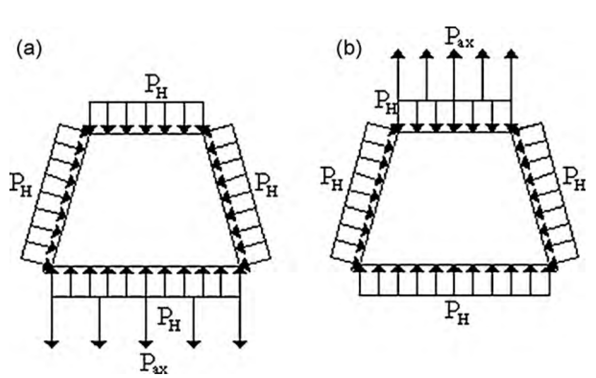

Fig. 2. The truncated conical shell under combined hydrostatic pressure and axial tension loads that applied to (a) the lower base and (b) the upper base.

3. Basic equations

It is assumed that the FGM is made of a mixture of a metal phase (denoted by "m") and a ceramic phase (denoted by "c"), with the material composition varying smoothly along its thickness direction (i.e. in the z-axis) only. According to rule of mixture, the effective Young's modulus and Poisson's ratio of FGMs can be written as (see Reddy and Chin, 1998; Shen, 2009)

$$E_{fg} = (E_c - E_m)V_c + E_m, \quad \nu_{fg} = (\nu_c - \nu_m)V_c + \nu_m \tag{5}$$

where E_m, ν_m and E_c, ν_c are the Young's modulus and Poisson's ratio of the metal and ceramic surfaces of the FGM conical shell, respectively. V_c is the ceramic volume fraction and we assume V_c follows a simple power law as (Pitakthapanaphong and Busso, 2002):

$$1.\ \text{Linear}: \quad V_c = \bar{z} + 0.5, \quad \bar{z} = \frac{z}{h} \tag{6}$$

$$2.\ \text{Quadratic}: \quad V_c = (\bar{z} + 0.5)^2 \tag{7}$$

$$3.\ \text{Inverse quadratic}: \quad V_c = 1 - (0.5 - \bar{z})^2 \tag{8}$$

The stress–strain relations for an FGM truncated conical shell are given as follows:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} e_1 - z\dfrac{\partial^2 w}{\partial S^2} \\ e_2 - z\left(\dfrac{1}{S^2}\dfrac{\partial^2 w}{\partial \varphi^2} + \dfrac{1}{S}\dfrac{\partial w}{\partial S}\right) \\ e_{12} - z\left(\dfrac{1}{S}\dfrac{\partial^2 w}{\partial S\,\partial \varphi} - \dfrac{1}{S^2}\dfrac{\partial w}{\partial \varphi}\right) \end{bmatrix} \tag{9}$$

where σ_1 and σ_2 are the stress components in the curvilinear coordinate directions S and θ on the reference surface, respectively. e_1 and e_2 are the strain components in the curvilinear coordinate directions S and θ on the reference surface, respectively. σ_{12} and e_{12} are the shear stress and shear strain. The quantities Q_{ij}, $i, j = 1, 2, 6$ for FGMs are

$$Q_{11} = Q_{22} = \frac{(E_c - E_m)V_c + E_m}{1 - [(\nu_c - \nu_m)V_c + \nu_m]^2}; \quad Q_{66} = \frac{(E_c - E_m)V_c + E_m}{1 + (\nu_c - \nu_m)V_c + \nu_m}$$
$$Q_{12} = \frac{[(E_c - E_m)V_c + E_m][(\nu_c - \nu_m)V_c + \nu_m]}{1 - [(\nu_c - \nu_m)V_c + \nu_m]^2}; \tag{10}$$

The force and moment resultants are expressed by

$$[(T_1, T_2, T_{12}), (M_1, M_2, M_{12})] = \int_{-h/2}^{h/2} (1,\ z)(\sigma_1, \sigma_2, \sigma_{12})dz \tag{11}$$

The relations between the forces and stress function Φ are given by