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Vehicle–snow interaction: Testing, modeling and validation for combined longitudinal and lateral slip

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Abstract

General operations of a vehicle involve simultaneous slip in the longitudinal and lateral directions, the combination of which is much more complicated than purely longitudinal or lateral motions. During vehicle–snow interactions, additional complexities arise due to uncertainties of snow material properties and of interfacial properties between the tire and snow, calling for the stochastic modeling of the interactions to validate the model. For validation, a statistical framework was formed with several components: a deterministic, physically-based tire–snow interaction model, a stochastic metamodel based on the physical model, a statistical model for calibration, prediction using the models, validation metrics, and new test data using an instrumented vehicle. The longitudinal and lateral drawbar pulls, and the torque and overturning moment, were used simultaneously to calibrate model parameters for a front and a rear tire. Four local and global validation metrics and extensive summary statistics were used to assess the quality of the models, with good results.

Keywords: Snow; Drawbar pull; Traction; Slip angle; Validation metrics; Calibration; Bayesian; Metamodel; Stochastic; Gaussian process

1. Introduction

The physics of a vehicle interacting with snow is complicated due to uncertainties of the mechanical properties of snow as well as of interfacial properties between snow and vehicle. Normal vehicle operations involve the longitudinal slip for straight-line motion and the combined longitudinal and lateral slip when a vehicle turns. Detailed studies of vehicle–snow interaction as a function of longitudinal slips have been conducted recently [1–3]. Although extensive modeling has been done for combined-slip situation [1,4–8], testing and validation of models have been lacking.

The formation of seasonal snow cover is subject to many environmental factors such as temperature, wind, and moisture. Consequently, variations of the properties of naturally-occurring snow are typically large necessitating a statistical approach. In addition, field conditions are more realistic and challenging than laboratory conditions. Consequently, the focus of our recent series of work [1–3,9,10], including this paper, is on characterizing the uncertainties of vehicle–terrain interaction for field conditions using a human driver as opposed to the traditional single-wheel soil-bin type of approach.

The uncertainties of snow properties for combined-slip conditions have been modeled using the metamodel approach in [5], and the polynomial chaos approach in [8]. There is, however, another type of uncertainty related to the various assumptions in the models themselves. These types of uncertainty were considered in [3,11] utilizing recent statistical framework for the validation of models [12,13]. The statistical framework consists of four stages: metamodel building, calibration, prediction and validation. Although the question of whether a model is validated is

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Nomenclature

$\tau_{mod}, \tau_{zx}, \tau_{zy}$	total, longitudinal, and lateral friction-limited shear stress at tire–snow interface
α	slip angle
$\bar{z}$	distance from the centroid of the lateral contact area to tire center
$\bar{\sigma}$	von Mises stress
β	friction angle for Drucker–Prager criterion
ϵ_v^p	volumetric plastic strain
$\mathbf{x}_{calibrated}$	calibrated parameters
μ	coefficient of Coulomb friction
$\boldsymbol{\Omega}$	covariance function of the statistical model
ω	angular velocity
ϕ	friction angle for Mohr–Coulomb criterion
σ_n	normal stress on tire
σ_y	normal stress on tire in the lateral direction
τ_Σ	shear stress at tire–snow interace
$\mathbf{x}$	parameters in the statistical model
θ, θ_0	anglular position, and exit angle of tire
A_y	contact area in the lateral direction
b	width of tire
c	cohesion for Mohr–Coulomb criterion
c_1, c_2, c_3	hardening constants of snow
CI	credible interval for Bayesian statistical model
D	diameter of tire
$E(t)$	mean error between model and test as a function of time
$f(\mathbf{x})$	target variables of the statistical model as a function of parameters $\mathbf{x}$
F_x, F_y, F_z	longitudinal drawbar pull, lateral drawbar pull (force), normal force on tire
f_{rr}	rolling resistance of tire itself
F_{zx}, F_{zy}	longitudinal, lateral traction
i_x	longitudinal slip
j_x, j_y, j_Σ	longitudinal, lateral, and total shear displacement
j_{y0}	maximum lateral shear displacement
K_{shear}	shear stress-shear displacement modulus
M_x	overturning moment
M_y	torque on tire
p	hydrostatic pressure
p_a, p_d	yield surface cap location, and cohesion for Drucker–Prager criterion
r	tire radius
R_x, R_y	longitudinal, and lateral motion resistance
v_x, v_y	longitudinal, and lateral velocity
Y	test data in the statistical model
z, z_0	sinkage, and maximum sinkage of tire
$\boldsymbol{\Omega}$	angular velocity of the vehicle
$\boldsymbol{r}_{t/a}$	position vector at the tire center relative to the GPS antenna
$\boldsymbol{v}_t, \boldsymbol{v}_a, \boldsymbol{v}_{t/a}$	velocity at tire center, GPS antenna, and relative velocity

often raised, the question of whether validation follows existing guidelines [14] is seldom raised. In addition, validation is often conducted with visual comparisons of test and model results without using statistical validation metrics [15]. It is highly desirable to assess the uncertainties of vehicle–terrain interaction using validation metrics within a rigorous but flexible statistical framework – statistical uncertainties call for statistical measures.

Reliable test data is an important component of the statistical framework. Recently, a test vehicle was developed for the validation of vehicle–terrain models [9], and it has been used in studying the longitudinal motion of a vehicle traversing snow [3] and soil [10]. The test vehicle is complemented by equipment for in-situ and laboratory tests of materials such that properties of materials can be used in the statistical methods.

While presenting new test data for combined slip, the purpose of this paper is to validate a simple, but physically-based, tire–snow interaction model via a statistical framework using four validation metrics. The paper is organized as follows. Snow material and indentation models, as well as tire–snow interaction model are presented in Section 2. The statistical framework is discussed in Section 3. Experimental procedures are discussed in Section 4. Results are given in Section 5, with discussion and conclusions given in Section 6.

2. Vehicle–snow interaction

2.1. Snow material model

To model the mechanical properties of snow, we use the Drucker–Prager (DP) plasticity model with a cap used previously in [4,16]. The pressure-sensitive yield criterion is defined as:

$$\bar{\sigma} - p\tan\beta - p_d = 0 \tag{1}$$

where β and p_d are the friction angle and cohesion, respectively; $\bar{\sigma}$ is the equivalent (von Mises) stress, and p is the hydrostatic pressure ($p = -\frac{\sigma_{kk}}{3}$), where repeated indices are summed. The hardening of the material is represented by the location of the cap p_a such that Eq. (1) is modified to be:

$$\bar{\sigma} - p_a\tan\beta - p_d = 0 \tag{2}$$

p_a is considered as a material parameter and is expressed as:

$$\log_{10}p_a = c_1 - c_2\exp(-\epsilon^p - c_3(\epsilon^p)^3) \tag{3}$$

where $\epsilon^p = \epsilon_{kk}^p = 3\epsilon_v^p$ is related to the volumetric plastic strain ϵ_v^p, c_1, c_2 and c_3 are constants; p_a is in MPa.

Drucker–Prager yield criterion can be related to the Mohr–Coulomb (MC) yield criterion using the following approximate relationship: