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An improved dynamic model for hub and laminated composite plate system considering warping effect

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ABSTRACT

In previous investigations of rigid-flexible coupling dynamics of the laminated plate, warping effect caused by the deformation of the normal vector of the mid-plane was neglected, which may lead to a significant error of the inter-laminar transverse shear stress. In this paper, an improved dynamic formulation for the rigid-flexible coupling dynamics of hub and laminated composite plate system is proposed based on the global–local higher-order shear deformation theory. By enforcing free shear traction conditions for both surfaces and continuity conditions of displacements and transverse shear stresses on the laminate interfaces, the dynamic equations for multi-body system are established based on the principle of virtual work. A numerical example for dynamic analysis of a hub and laminated composite plate system is presented to show different results obtained by different displacement theories. It is demonstrated that the results obtained by the present formulation satisfy both the continuity conditions of inter-laminar shear stresses and the free shear traction conditions of upper and lower surfaces.

1. Introduction

Due to the characteristics of high strength and high modulus, composite materials have been widely used in industrial areas, especially in the field of aircraft and spacecraft.

Based on the Kirchhoff assumption, the classical laminated plate theory is only proper for very thin laminated composite plates due to the neglect of the effects of transverse shear strains. The first-order shear deformation theory (FSDT) [1,2] was attempted to take into account the effects of the transverse shear strain. However, the accuracy of the results obtained by the FSDT is mainly dependent on the shear correction factors, such that the shear stress results cannot be accurately calculated by using FSDT.

In order to overcome the limitation of the classical laminated plate theory and the first-order shear deformation theory, the global higher-order plate theory (HSDT) [3] was developed. However, this theory is not of the layer-wise type, which does not satisfy the continuity conditions of transverse shear stresses between layers, and then a number of layer-wise plate models have been developed, which represent the zig-zag behavior of the in-plane displacement through the thickness. Di Sciuva [4] proposed a zig-zag theory which considered the continuity conditions of transverse shear stresses between layers, but the free conditions of the upper and lower surfaces are not satisfied. Then Di Sciuva [5,6] proposed a third order zig-zag theory, in which both the continuity conditions of transverse shear stresses between layers and the free shear traction conditions for upper and lower surfaces are imposed. However, the theory does not satisfy the accuracy requirement for thick composite plate. Cho [7–9] modified the zig-zag theory and considered the transverse normal strain in the thickness direction. Kapuria [10] extended the high-order zig-zag theory to the dynamic analysis of composite beam. Because the number of variables depends on the number of layers, they are not computationally efficient for laminated plates with large number of layers.

The displacements mode of global–local higher-order shear deformation theory is composed of both global displacements and local displacements. By enforcing free shear traction conditions for both surfaces and continuity conditions of displacements and transverse shear stresses on the laminate interfaces, the number of unknowns can be reduced. Li and Liu [11] proposed a modified displacement theory which considered both the continuity shear stress condition and the free conditions of the upper and lower surfaces by leading into local displacement. Wu [12–15] extended the investigation to arbitrary layered composite laminates. Recently, Han and Bauchau [16] performed a semi-discretization of the general equations of three-dimensional nonlinear elasticity for static analysis of the laminated shell. However, these formulations are

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mostly applied to static analysis of laminated composite shell structure.

In this paper, an improved dynamic model for hub and laminated composite plate system is proposed based on global–local higher-order 1,2–3 shear deformation theory. The continuity conditions of in-plane deformation and inter-laminar transverse shear stress as well as the free shear traction conditions for both surfaces are satisfied. The number of the independent variables of each node is reduced to 13, which does not change with the number of plies. The displacement mode is then developed in rigid-flexible coupling dynamics for a hub and composite plate system, based on the hybrid coordinate formulation. By using the principle of virtual work, the dynamic equations for multi-body system are established. A simulation example of a hub and laminated composite plate system is presented to show different results obtained by the first-order shear deformation theory (FSDT), the global higher-order plate theory (HSDT) and the improved dynamic model based on global–local higher-order 1,2–3 shear deformation theory.

2. Global–local 1,2–3 high-order displacement theory

The dynamic theory is based on the following assumptions:

(1) The normal stress in the thickness direction is neglected.
(2) The relative slide between each layer is not taken into account.
(3) The composite plate satisfies the free condition of the upper and lower surface.

The global–local high-order displacement mode is given by [17]

$$u^k = u_G + u_L^k, \quad v^k = v_G + v_L^k, \quad w^k = w_G \tag{1}$$

where the global displacement can be expressed as

$$\begin{aligned} u_G &= u_0(x,y) + zu_1(x,y) + z^2u_2(x,y) + z^3u_3(x,y) \\ v_G &= v_0(x,y) + zv_1(x,y) + z^2v_2(x,y) + z^3v_3(x,y) \\ w_G &= w_0(x,y) \end{aligned} \tag{2}$$

which is the same as Reddy's higher order deformation theory. Fixing items are then introduced to describe the warping effect, and the local displacement takes the form:

$$\begin{aligned} u_L^k &= \bar{u}_L^k(x,y,z) + \hat{u}_L^k(x,y,z) \\ v_L^k &= \bar{v}_L^k(x,y,z) + \hat{v}_L^k(x,y,z) \\ \bar{u}_L^k &= \xi_k u_1^k + \xi_k^2 u_2^k, \quad \hat{u}_L^k = \xi_k^3 u_3^k \\ \bar{v}_L^k &= \xi_k v_1^k + \xi_k^2 v_2^k, \quad \hat{v}_L^k = \xi_k^3 v_3^k \end{aligned} \tag{3}$$

where $\xi_k = a_k z - b_k$, $a_k = 2/(z_{k+1} - z_k)$, $b_k = (z_{k+1} + z_k)/(z_{k+1} - z_k)$, $k = 1, 2, \ldots, n$, in which Li and Liu [17] certified that the zeroth-order term is the mid-plane displacement of each composite layer, which can be omitted from layer-dependent displacement assumption since the continuity conditions have been satisfied on the laminate interfaces, resulting in the uselessness of assuming the zeroth-order term. The first order term is used to described rotation angle, and the second order term represents the curvature of the displacement distribution of the composite laminate, while the third-order term can be associated with the curvature of the transverse stress distribution. They are equally important to composite performance, especially when a laminate has a large number of layers.

The continuity conditions of the inter-laminar in-plane displacement take the form

$$\begin{aligned} \bar{u}_L^k(x,y,z_k) &= \bar{u}_L^{k-1}(x,y,z_k) \\ \hat{u}_L^k(x,y,z_k) &= \hat{u}_L^{k-1}(x,y,z_k) \\ \bar{v}_L^k(x,y,z_k) &= \bar{v}_L^{k-1}(x,y,z_k) \\ \hat{v}_L^k(x,y,z_k) &= \hat{v}_L^{k-1}(x,y,z_k) \end{aligned} \tag{4}$$

The continuity conditions of the inter-laminar shear stresses take the form

$$\begin{aligned} \tau_{xz}^k(x,y,z_k) &= \tau_{xz}^{k-1}(x,y,z_k) \\ \tau_{yz}^k(x,y,z_k) &= \tau_{yz}^{k-1}(x,y,z_k) \end{aligned} \tag{5}$$

where $\tau_{yz}^k = Q_{44k}\gamma_{yz}^k + Q_{45k}\gamma_{xz}^k$, $\tau_{xz}^k = Q_{45k}\gamma_{yz}^k + Q_{55k}\gamma_{xz}^k$, and Q_{44k}, Q_{45k}, Q_{55k} are the stiffness coefficients of the kth layer, and γ_{xz}^k, γ_{yz}^k represent the transverse shear strains, which are given by $\gamma_{yz}^k = \partial v^k/\partial z + \partial w^k/\partial y$, $\gamma_{xz}^k = \partial u^k/\partial z + \partial w^k/\partial x$.

The free conditions of the lower and upper surfaces can be written as

$$\begin{aligned} &\tau_{yz}^1(x,y,z_1) = 0, \quad \tau_{xz}^1(x,y,z_1) = 0 \\ &\tau_{yz}^n(x,y,z_{n+1}) = 0, \quad \tau_{xz}^n(x,y,z_{n+1}) = 0 \end{aligned} \tag{6}$$

which lead to $\gamma_{xz}^1(x,y,z_1) = 0$, $\gamma_{yz}^1(x,y,z_1) = 0$ and $\gamma_{xz}^n(x,y,z_{n+1}) = 0$, $\gamma_{yz}^n(x,y,z_{n+1}) = 0$.

Satisfying the continuity conditions of the inter-laminar in-plane displacements and transverse shear stresses as well as the free shear traction conditions for both surfaces, the displacement of the kth layer is given by

$$\begin{aligned} u^k &= u_0 + \varphi_1^k u_1^1 + \varphi_2^k u_1 + \varphi_3^k u_2 + \varphi_4^k u_3 + \varphi_5^k w_{0,x} + \varphi_6^k v_1^1 \\ &\quad + \varphi_7^k v_1 + \varphi_8^k v_2 + \varphi_9^k v_3 + \varphi_{10}^k w_{0,y} \\ v^k &= v_0 + \psi_1^k u_1^1 + \psi_2^k u_1 + \psi_3^k u_2 + \psi_4^k u_3 + \psi_5^k w_{0,x} + \psi_6^k v_1^1 \\ &\quad + \psi_7^k v_1 + \psi_8^k v_2 + \psi_9^k v_3 + \psi_{10}^k w_{0,y} \\ w^k &= w_G = w_0 \end{aligned} \tag{7}$$

where

$$\varphi_i^k = \xi_k R_i^k + \xi_k^2 S_i^k + \xi_k^3 T_i^k + Z_i \tag{8}$$

$$\psi_i^k = \xi_k O_i^k + \xi_k^2 P_i^k + \xi_k^3 Q_i^k + ZZ_i \quad i = 1, 2, \ldots, 10 \tag{9}$$

and the expression of R_i^k, S_i^k, T_i^k, O_i^k, P_i^k, Q_i^k, Z_i, ZZ_i are given in Ref. [12].

3. Variational dynamic equations of a laminated composite plate

Finite element method is used for discretization. Assuming that

$$u_k = \boldsymbol{S}_{k+1}\boldsymbol{p}_e, \quad v_k = \boldsymbol{S}_{k+5}\boldsymbol{p}_e, \quad k = 0, 1, 2, 3,$$

$$u_1^1 = \boldsymbol{S}_9\boldsymbol{p}_e, \quad v_1^1 = \boldsymbol{S}_{10}\boldsymbol{p}_e,$$

$$w_0 = \boldsymbol{S}_{11}\boldsymbol{p}_e, \quad w_{0,x} = \boldsymbol{S}_{12}\boldsymbol{p}_e, \quad w_{0,y} = \boldsymbol{S}_{13}\boldsymbol{p}_e,$$

$$\boldsymbol{S}_{12} = \partial\boldsymbol{S}_{11}/\partial x, \quad \boldsymbol{S}_{13} = \partial\boldsymbol{S}_{12}/\partial y,$$