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A shape memory alloy model by a second order phase transition

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ABSTRACT

Motivated by the experimental results of the paper [1] and unlike the general theories of shape memory alloys (SMAs), in this paper we suggest for such materials a phase field model by a second order phase transition. So that, with this new system we obtain a simulation of phase dynamics very convenient to describe the natural behavior of these materials. The differential system is governed by the motion equation, the heat equation and the Ginzburg–Landau (GL) equation and by a constitutive law between the phase field, the temperature, the strain and the stress. The use of this new model is characterized by new potentials of the GL equation and by a new dependence on the temperature in the constitutive equation. Using this new model, we obtain simulations in better agreement with experimental data and respect to previous work [2].

1. Introduction

The shape memory alloys (SMAs) are metal alloys that can recover the original strain, when heated over a critical temperature (see [3–8]). The SMAs show two phases, called austenite and martensite, moreover the martensite presents two possible internal structures, twinned and detwinned, as shown in Fig. 1.

For small values of the stress, under a cooling, we observe a reversible transformation from austenite to twinned martensite. When the stress is high enough, the same variation of temperature produces a phase transition from austenite to detwinned martensite and vice versa (see [5,9–11]). Moreover, if the temperature is fixed, an increasing of the stress can realize a phase transition from austenite to martensite and vice versa. These behaviors are well represented by the one-dimensional phase diagram of Fig. 2.

Another meaningful behavior of the SMAs is well described in Fig. 3, where the curves in the stress-strain diagrams assume meaningful changes as a function of temperature.

In this paper, as in [2,12], we suggest a model making use of the Ginzburg–Landau equation and of a suitable constitutive equation between the phase field φ, the stress $\boldsymbol{\sigma}$, and the strain $\boldsymbol{\varepsilon}$. Unlike previous researches, the coefficients of the constitutive equation depend on temperature and the austenite–martensite phase transition considered is of second order. In the experimental work of Darling et al. [1] the alloy AuZn is studied. They found that this alloy exhibits the shape-memory effect and a phase transition at 64.75 K that appears to be continuous (second-order) from the specific heat data.

This new model makes the differential system more easily manageable, because it uses potentials of lower degree than the previous ones. Moreover, any production of latent heat does not appear evident in the austenite–martensite transition. Conversely, if we observe the graph of Fig. 4, the transition from austenite to martensite occurs within a strip of stress or temperature, whereby the transition cannot be of first order, but because it occurs by a continuous transformation inside finite intervals of temperature or stress, it seems reasonable to describe the phenomenon by a second order phase transition.

2. Mathematical model. Austenite and martensite

We formulate a theory based on the Ginzburg–Landau theory of phase transitions using the notion of order parameter φ (see [2,12–14]).

The structure order balance law in local form can be formulated as

$$\rho s = \nabla \cdot \mathbf{p}, \tag{1}$$

where s is the specific internal structure order per unit mass, $\mathbf{p}$ denotes the structure order flux and ρ is the reference mass density.

We assume the following constitutive equations for s and $\mathbf{p}$

$$s = \gamma\dot{\varphi} + \theta_c F'(\varphi) + (\hat{\theta}_c - \alpha|\boldsymbol{\sigma}|)G'(\varphi), \tag{2}$$

$$\mathbf{p} = \rho\kappa\nabla\varphi, \tag{3}$$

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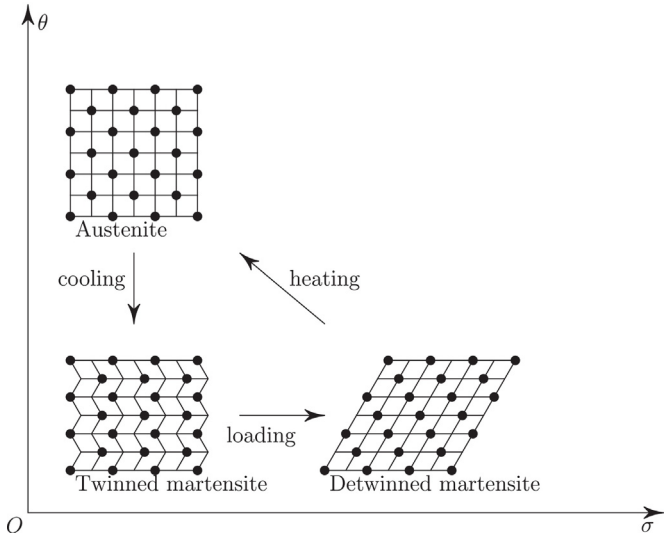

Fig. 1. Transitions between austenite, twinned martensite and detwinned martensite, based on changes in stress and temperature.

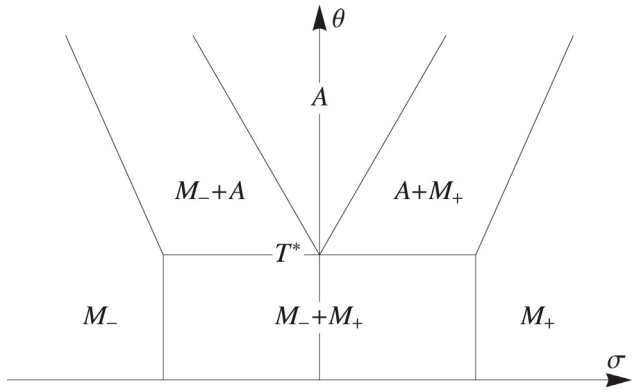

Fig. 2. In this graphic M_+ and M_- denote the detwinned martensite, while $M_+ + M_-$ represents the zone of twinned martensite. Finally, the austenite is represented by the zone A.

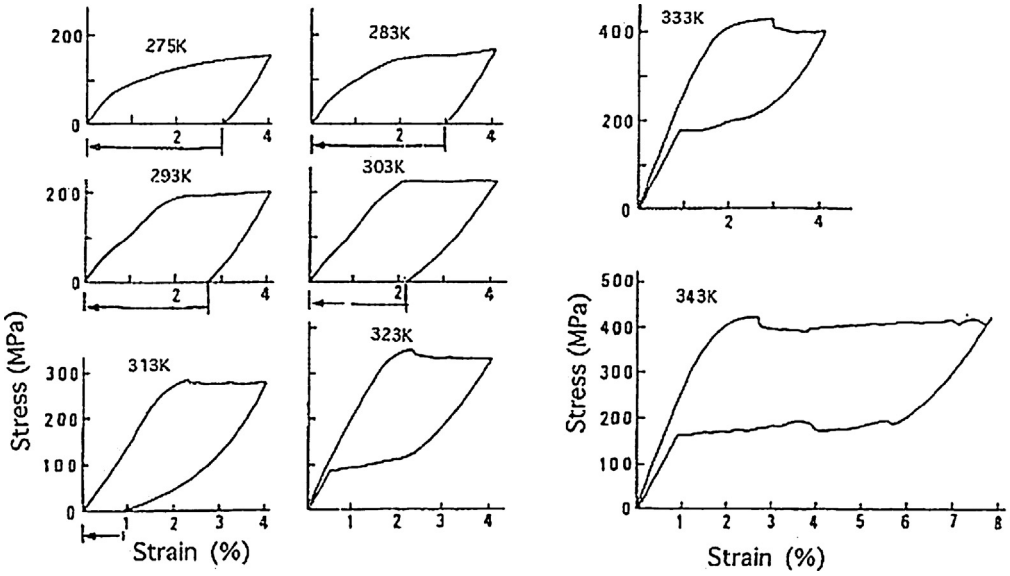

Fig. 3. NiTi stress–strain curves at different temperatures. Curtesy Subari [8].

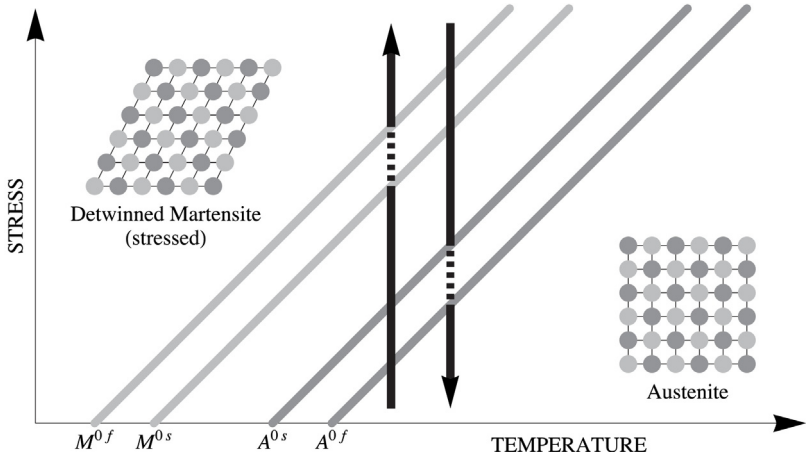

Fig. 4. In this graphic, we have two vertical lines at a fixed temperature. When, from austenite phase, we increase the stress, the phase transition begins only when it reaches the line M^{0s}. Otherwise, if from the martensite phase, we decrease the stress, then the transition begins only when the stress reaches the line A^{0s}. In both cases, the transition takes place only after a further increase or decrease of stress.