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Investigation of guided waves propagation in orthotropic viscoelastic carbon–epoxy plate by Legendre polynomial method

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ABSTRACT

The effect of viscoelasticity on the guided waves propagation in viscoelastic plate has been investigated according to multi-aspect. To this purpose, an extension of the Legendre polynomial method is proposed to formulate the guided waves equation in orthotropic viscoelastic plate composed of carbon–epoxy. The validity of the proposed Legendre polynomial method is illustrated by comparison with available data. The convergence of the method is discussed through a numerical example. The hysteretic and Kelvin–Voigt viscoelastic models are used to integrate the imaginary part of the complex stiffness matrix associated with the viscoelastic plate in this study. Accordingly, both viscoelastic models do not affect on the dispersion curves results. However, appreciable effects are seen in the attenuation curves. Also, the sensitivity of the guided waves propagation caused by variations of elastic and viscoelastic modulus has been studied in detail. Finally, the advantages of the Legendre polynomial method are described.

1. Introduction

The guided acoustic technique has been widely used for evaluating different structures [1]. In fact, these acoustic waves are so important for non-destructive testing (NDT) in the industry. Accordingly, these waves are represented a means motivated for detecting flaws because of their effectiveness for quickly testing a fairly long extended series of the specimen as shown by experimental and numerical works. Therefore, there are extensive literatures in which the authors take into account the propagation of guided waves in various elastic media. Notwithstanding the efforts were devoted to study the acoustic wave propagation in the elastic media, those studies remain more and less worthwhile in the case of real material. Consequently, with rapid development of material science, there are solids media take account of attenuation and dispersion phenomena related to the viscoelastic nature of the material with their increasing usage in civil engineering, in the automotive or aerospace industries since they combine low weight, high specific strength and excellent possibilities to absorb energy [2]. The difficulties associated with the rheological model to integrate the imaginary part of the complex stiffness matrix associated with the viscoelastic plate are doubled by the difficulties in multiple modal analysis of dispersive character.

Many researchers have been conducted in the area of viscoelastic material characterization [3,4]. Moreover, Simonetti [5], investigated the propagation of Lamb wave in elastic plates coated with viscoelastic materials and considered the viscoelastic coating effect on the dispersion properties of Lamb wave propagation in elastic plates. In same context, Sun et al. [6] had developed the effect of material viscoelasticity on the propagation of Lamb wave in a viscoelastic thin plate. These results motivate our interest in the effect of a general state of material viscoelasticity on the dispersion behavior of guided waves.

Many methods have been proposed to study the dispersion and attenuation curves of guided waves in viscoelastic structures. As early in 2008, Ben Amor and Ben Ghozlen [7] had developed a Stiffness Matrix method to investigate the guided modes modeling in viscoelastic multilayered composites. Then, in 2011, Haïat et al. [8] had developed a Finite Element method to investigate the lateral wave with axial transmission in viscoelastic cortical bone. Mazzotti et al. [9] had used the Semi Analytical Finite Element (SAFE) method to analyze the guided waves dispersion for pre-stressed viscoelastic waveguides. Same method is used by Bartoli et al. [10], to a detailed study of guided wave propagation in different viscoelastic structures. Torres-Arredondo and Fritzen [11] had developed

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higher order plate theory method to investigate the phase velocity and attenuation in orthotropic viscoelastic plate.

Multivariate research in the viscoelastic plane by these matrix methods can be limited [11]. Consequently, these methods may have difficulties in properly output the phase velocity and attenuation curves.

At the end to avoid the drawbacks and specifically the problem to solve dispersion equation, a method based on an expansion polynomial has been developed. In 1999, Lefebvre et al. [12] developed the Legendre polynomial method to investigate the guided waves in piezoelectric plates. Thereafter, this method is widely used to simulate the guided waves.

The Legendre polynomial method has the specificity to incorporate automatically the boundary conditions into the constitutive equations by the rectangular window function. Moreover, to find the characteristics of guided waves, converged results are obtained.

The present study extends the Legendre polynomial method for modeling dispersive solutions .The carbon–epoxy has been retained to illustrate the response of a viscoelastic plate. Both hysteretic and Kelvin–Voigt viscoelastic models were used in this study. Accordingly, both viscoelastic models do not affect the dispersion curves results. However, appreciable effects are seen in the attenuation curves. Thereafter, this work includes numerical examples which illustrate the effects of elastic and viscoelastic modulus on dispersion and attenuation curves. Finally, the advantages of the Legendre polynomial method are described.

2. Formulation of the problem

Consider an orthotropic viscoelastic plate which is infinite horizontally with a thickness $h = 1$ mm. Homogeneous structure was chosen because of its highly anisotropic character. This plate is characterized by its complex stiffness tensor C^*_{ijkl} and mass density ρ. This example was fully studied in References [10,11,13]. The elastic and viscoelastic material properties are given in Table 1. The material was characterized at 2.242 MHz by the use of ultrasonic waves transmitted through a plate-shaped sample immersed in water as mentioned in Reference [11]. The plate described in Cartesian coordinate system(x_1, x_2, x_3), occupies the region $0 \le x_3 \le h$, as shown in Fig. 1.

On the assumption that the guided waves propagating in the x_1 direction, the total displacements are expressed as

$$u_i = u_i(x_1, x_3, t) \quad i = 1, 2, 3 \tag{1}$$

Table 1
Properties of unidirectional carbon–epoxy 1 mm thick plate with symmetry orthotropic [11].

Property	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{44}	C_{55}	C_{66}
Carbon–epoxy	132	6.9	12.3	5.9	5.5	12.1	3.32	6.21	6.15
Property	η_{11}	η_{12}	η_{13}	η_{22}	η_{23}	η_{33}	η_{44}	η_{55}	η_{66}
Carbon–epoxy	0.4	0.001	0.016	0.037	0.021	0.043	0.009	0.015	0.02

C_{ijkl}(Gpa), η_{ijkl}(Gpa), $\rho = 1560(\mathrm{kg/m^3})$.

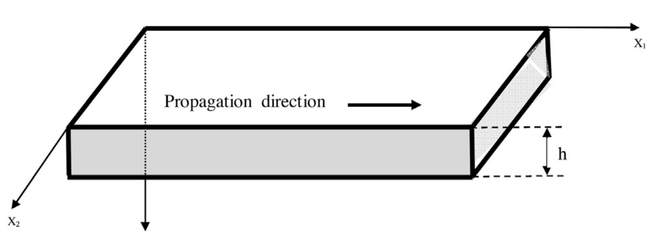

Fig. 1. The model of the homogeneous viscoelastic plate.

where u_i is the elastic displacements.

Thus, the dynamic equation for the viscoelastic plate for the guided waves is governed by,

$$\begin{aligned} \rho\frac{\partial^2 u_1}{\partial t^2} &= \frac{\partial T_{11}}{\partial x_1} + \frac{\partial T_{13}}{\partial x_3} \\ \rho\frac{\partial^2 u_2}{\partial t^2} &= \frac{\partial T_{12}}{\partial x_1} + \frac{\partial T_{23}}{\partial x_3}, \\ \rho\frac{\partial^2 u_3}{\partial t^2} &= \frac{\partial T_{13}}{\partial x_1} + \frac{\partial T_{33}}{\partial x_3} \end{aligned} \tag{2}$$

with ρ being the density of the material and T_{ij} is stress.

The boundary conditions for an orthotropic viscoelastic plate generally require that: the component of the stress should be zero at the upper and bottom surfaces.

By introducing the rectangular window function $\pi_{0,h}(x_3)$ [12],

$$\pi_{0,h}(x_3) = \begin{cases} 1 & 0 \le x_3 \le h \\ 0 & \text{otherwise} \end{cases}$$

the above-mentioned boundary conditions ($T_{13} = T_{23} = T_{33} = 0$ at $x_3 = 0$ and $x_3 = h$) are automatically incorporated in the constitutive relations,

$$\begin{aligned} T_{11} &= C^*_{11}\frac{\partial u_1}{\partial x_1} + C^*_{13}\frac{\partial u_3}{\partial x_3}, T_{12} = C^*_{66}\frac{\partial u_2}{\partial x_1} \\ T_{13} &= (C^*_{55}\frac{\partial u_3}{\partial x_1} + C^*_{55}\frac{\partial u_1}{\partial x_3})\pi_{0,h}, \\ T_{23} &= (C^*_{44}\frac{\partial u_2}{\partial x_3})\pi_{0,h}, \\ T_{33} &= (C^*_{13}\frac{\partial u_1}{\partial x_1} + C^*_{33}\frac{\partial u_3}{\partial x_3})\pi_{0,h} \end{aligned} \tag{3}$$

The complex components of the stiffness matrix C^*_{ijkl} according to the hysteretic model [14] are independent of frequency, thus:

$$C^*_{ijkl} = C_{ijkl} + i\eta_{ijkl} \tag{4}$$

In the Kelvin–Voigt model [14], the complex component of the stiffness matrix is dependent of frequency, thus:

$$C^*_{ijkl} = C_{ijkl} + i\left(\frac{f}{\tilde{f}}\right)\eta_{ijkl} \tag{5}$$

C_{ijkl} and η_{ijkl} are elastic and viscous coefficients respectively, f is the frequency and $\tilde{f}$ is the frequency of characterization equal to 2.242 MHz.

For a wave being propagated in the x_1 direction in the plate, we assume that the displacement components to be on the form:

$$u_i(x_1, x_2, x_3, t) = U_i(x_3)\exp(ikx_1 - i\omega t), \tag{6}$$

$U_i(x_3)$ represents the amplitude of displacements in the x_1, x_2 and x_3 directions, k is the magnitude of the wave vector in the propagation direction.

Substituting Eqs. (3) and (6) into Eq. (2) gives:

$$\begin{aligned} \omega^2 U_1 = &-k^2\frac{C^*_{11}}{\rho}U_1 + ik(\frac{C^*_{13}+C^*_{55}}{\rho})U'_3 + \frac{C^*_{55}}{\rho}U_1{}'' \\ &+ ik\left(\frac{C^*_{55}}{\rho}\pi_{0,h}(x_3)\right)'U_3 + \left(\frac{C^*_{55}}{\rho}\pi_{0,h}(x_3)\right)'U'_1 \end{aligned} \tag{7a}$$

$$\omega^2 U_2 = -k^2\frac{C^*_{66}}{\rho}U_2 + \frac{C^*_{44}}{\rho}U_2{}'' + \left(\frac{C^*_{44}}{\rho}\pi_{0,h}(x_3)\right)'U'_2 \tag{7b}$$