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Size effect on nominal flexural strength of concrete beams influenced by damage gradient

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ABSTRACT

Based on the first-order gradient damage theory where a damage gradient $D_{,m}$ and internal characteristic length parameter l_m were introduced into the constitutive equations, an approach is proposed to consider the influence of damage gradients on the size effect of concrete beams. In the numerical implementation of the first-order gradient damage theory, damage values at Gauss points are calculated in each iterative step of non-linear finite element analysis, then the damage gradients of Gauss points are calculated by using the finite difference method. Geometrically similar unnotched pure bending concrete beams with a fixed ratio 4 of length to depth are simulated. The results show that the nominal flexural strength M_{nom} increases linearly with the internal characteristic length parameter l_m and decreases monotonically with the beam depth d. When l_m equals to zero, the nominal flexural strength M_{nom} becomes a constant. Otherwise, the size effect markedly increases with the internal characteristic length parameter l_m. A gradient damage size effect law (GDSEL) is proposed to predict the size effect of unnotched concrete beams. When the beam depth $d \to \infty$, the GDSEL produces a horizontal asymptote in the plot of M_{nom} versus d.

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1. Introduction

The size effect is a well known phenomenon in materials, such as metal [15], concrete [3,28], rock [13,28] and bone [9,10]. There are a number of experimental and theoretical studies [3,6,7,19,20,24,28] that confirm the existence of the size effect in concrete. Since large structures are often beyond the range of testing in laboratories, their design has to rely on a realistic extrapolation of testing results with smaller sizes. Civil engineers must extrapolate experimental outcomes at laboratory scale to the results which can be used in large scale situations. So, the physical understanding of size effects is of major importance.

Some researchers developed theories trying to 'predict' the size effect for scale ranges which cannot be tested in laboratory. There are two aspects of size effect: (1) Statistical and (2) Deterministic. The first statistical theory was introduced by Weibulll [24] (also called the weakest link theory) which postulates that a structure is as strong as its weakest component. The Weibull's size effect model is a power law for large structures that fails as soon as a macroscopic fracture initiates in one small material element. Since the stress redistribution is not considered, the structure fails when its strength is exceeded at the weakest spot. Therefore, it is unable to account for a spatial correlation between local material properties, and it does not include any characteristic length of micro-structures so that the deterministic size effect is ignored. Currently, there are two different deterministic theories of size effect, for concrete: the multi-fractal scaling law (MFSL) [1–3,12] and the Bažant's size effect law [26,27]. The fundamental assumption in the multi-fractal damage theory is that the material has perfect homogeneity when structure size $d \to 0$. The corresponding size effect law is of the form:

$$\sigma_N(d) = \left(A_m + \frac{B_m}{d}\right)^{1/2} \tag{1}$$

where σ_N is nominal strength, d is the external size of the structure, A_m and B_m are two constants obtained by fitting test or calculated data. In the fractal approach, σ_N decreases in a hyperbolic form with increasing d. The MFSL behaviour in the bilogarithmic plane $\ln \sigma_N$ versus $\ln d$ is non-linear and shows two asymptotes with slope $-1/2$ for small structures ($\lim_{d\to 0} \sigma_N = +\infty$) and slope zero for the largest ones ($\lim_{d\to\infty} \sigma_N = \sqrt{A}$), respectively.

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Two size effect laws proposed by Bažant for geometrically similar structures and the size effect laws are of the form:

$$\text{type 1}: \quad \sigma_N(d) = f_r^{\infty}\left(1 + \frac{rD_b}{d}\right)^{1/r} \text{ unnotched struct.} \tag{2}$$

$$\text{type 2}: \quad \sigma_N(d) = \frac{B_b f_t}{\sqrt{1 + d/d_0}} \quad \text{notched struct.} \tag{3}$$

where f_r^{∞} represents the elastic-brittle strength of concrete, r controls the curvature and shape of the law, D_b is the deterministic characteristic length having the meaning of the thickness of the cracked layer (if $D_b = 0$, the behavior is elastic–brittle), f_t denotes the tensile strength, B_b (depending on the geometry of the structure and crack) and d_0 are two unknown empirical constants to be determined. In the first type (unnotched structures), the maximum load is reached as soon as a macroscopic crack initiates. In the second type (notched structures), cracks grow in a stable manner prior to the maximum load. The material strength is bound for small sizes by a plasticity limit whereas for large sizes the material follows the linear elastic fracture mechanics.

In spite of many experiments exhibiting the size effect for noticed concrete specimens, the size effect theories are still all at empirical levels. In order to make clear what sources cause the size effect and how they affect the member behaviors, the numerical simulations are performed to investigate the size effect. Some remarkable works have been published in the 1990s. For example, statistical analyses were carried out with spatially correlated homogeneous distributions of tensile strength which were assumed to be random [8]. Carmeliet [12] combined a simple nonlocal damage model within a single finite element computational model, and studied two different length parameters: the characteristic length of the non-local damage model, and the correlation distance for the random field. The size effects were also simulated by many researchers [11,16,18,21,25,29].

We attempt to propose an approach based on the first-order gradient damage theory to capture the influence of damage gradients on the size effect. The outline of the paper is as follows. In Section 2, the first-order gradient damage theory and damage evolution of concrete are introduced. In Section 3, a numerical implementation of the first-order gradient damage theory is represented. In Section 4, the results of size effects in concrete beams with different prescribed parameters are analyzed. And, a gradient damage size effect law (GDSEL) for unnotched concrete beams is proposed. In Section 5, the difference between the GDSEL and the existing size effect laws is compared. The main conclusions are drawn in Section 6.

2. The first-order gradient damage theory and damage evolution of concrete

2.1. The first-order gradient damage theory

In order to describe the interaction of microstructures in a material, the first-order gradient damage theory was proposed by Zhao et al. [5]. In this theory, the strain tensor ε_{ij}, the scalar damage variable D and the damage gradient $D_{,m}$ ($D_{,m} = \partial D/\partial m\,(m = x, y, z)$) served as the state variables of the Helmholtz free energy per unit volume Ψ:

$$\Psi = \Psi(\varepsilon_{ij}, D, D_{,m}) \tag{4}$$

$$\dot{\Psi} = \frac{\partial \Psi}{\partial \varepsilon_{ij}}\dot{\varepsilon}_{ij} + \frac{\partial \Psi}{\partial D}\dot{D} + \frac{\partial \Psi}{\partial D_{,m}}\dot{D}_{,m} \tag{5}$$

For an isothermal and infinitesimal deformation process, the Clausius-Duhem inequation is

$$\sigma_{ij}\dot{\varepsilon}_{ij} - \dot{\Psi} \geq 0 \tag{6}$$

where σ_{ij} is the Cauchy stress tensor.

The substitution of (5) into the inequality (6) yields:

$$\left(\sigma_{ij} - \frac{\partial \Psi}{\partial \varepsilon_{ij}}\right)\dot{\varepsilon}_{ij} - \left(\frac{\partial \Psi}{\partial D}\dot{D} + \frac{\partial \Psi}{\partial D_{,m}}\dot{D}_{,m}\right) \geq 0 \tag{7}$$

The inequality (7) holds for an arbitrary value of $\dot{\varepsilon}_{ij}$, which requires

$$\sigma_{ij} - \frac{\partial \Psi}{\partial \varepsilon_{ij}} = 0 \tag{8}$$

$$-\left(\frac{\partial \Psi}{\partial D}\dot{D} + \frac{\partial \Psi}{\partial D_{,m}}\dot{D}_{,m}\right) \geq 0 \tag{9}$$

Defining internal characteristic length parameters as:

$$\frac{\partial D}{\partial D_{,m}} = l_m \quad (m = x, y, z) \tag{10}$$

where l_x, l_y, l_z are internal characteristic length parameters on the direction of x, y, z and have dimension of length. Obviously, when $l_x = l_y = l_z$, the damage model becomes an isotropic gradient damage model. When $l_x \neq l_y \neq l_z$, it is an anisotropic gradient damage model.

The initial state of material can be assumed that: $\varepsilon_{ij} = 0$, $\sigma_{ij} = 0$, $D = 0$, $D_{,m} = 0$, $\Psi_0 = 0$. The Helmholtz free energy Ψ is expanded to Taylor's series. The series is truncated at the second power of ε_{ij}, the Nth power of D and the first power of $D_{,m}$. Since ε_{ij} is an infinitesimal variable and D is a variable with a finite value ($0 \leq D \leq 1$). For the elastic isotropic damage, the expansion of Ψ is:

$$\Psi = \sum_{r=1}^{N} C^{(r)}D^r + \sum_{r=1}^{N} F_{ij}^{(r)}D^r\varepsilon_{ij} + D_{,m}M_{ijm}\varepsilon_{ij} + D_{,m}\sum_{r=1}^{N} H_{ijm}^{(r)}D^r\varepsilon_{ij} + \frac{1}{2}C_{ijkl}\varepsilon_{ij}\varepsilon_{kl}$$
$$+\frac{1}{2}\sum_{r=1}^{N} A_{ijkl}^{(r)}D^r\varepsilon_{ij}\varepsilon_{kl} + \frac{1}{2}D_{,m}\sum_{r=0}^{N} B_{ijklm}^{(r)}D^r\varepsilon_{ij}\varepsilon_{kl} + D_{,m}\sum_{r=1}^{N} P_m^{(r)}D^r \tag{11}$$

where $r = 1, 2\ldots N$. Note that the free energy density function Ψ is a scalar-valued function. Therefore, the coefficients $C^{(r)}$, ${P_m}^{(r)}$ in Eq. (11) should be the scalar coefficients since the damage variable D is a scalar variable. Because of the same reason, the coefficients ${P_m}^{(r)}$ should be the vector, in addition, ${F_{ij}}^{(r)}$ should be the second-order tensor coefficients, M_{ijm} and ${H_{ijm}}^{(r)}$ should be the third-order tensor, ${A_{ijkl}}^{(r)}$ and C_{ijkl} are the fourth-order tensors, ${B_{ijklm}}^{(r)}$ is the fifth-order tensor. So the products can have the scalar values according to the Einstein's summation convention.

The substitution of Eq. (11) into Eq. (8) yields:

$$\sigma_{ij} = \sum_{r=0}^{N} F_{ij}^{(r)}D^r + D_{,m}M_{ijm} + D_{,m}\sum_{r=1}^{N} H_{ijm}^{(r)}D^r$$
$$+\left\{C_{ijkl} + \sum_{r=1}^{N} A_{ijkl}^{(r)}D^r + D_{,m}\sum_{r=0}^{N} B_{ijklm}^{(r)}D^r\right\}\varepsilon_{kl} \tag{12}$$

When the damaged material is unloaded completely to the initial state, it is seen that $\varepsilon_{ij} = 0$, $\sigma_{ij} = 0$. Considering the irreversibility of damage, $D \neq 0$, $D_{,m} \neq 0$, from Eq. (12), it is found that:

$$\sum_{r=0}^{N} F_{ij}^{(r)}D^r + D_{,m}M_{ijm} + D_{,m}\sum_{r=1}^{N} H_{ijm}^{(r)}D^r = 0 \tag{13}$$

The substitution of Eq. (13) into Eq. (12) yields

$$\sigma_{ij} = \left\{C_{ijkl} + \sum_{r=1}^{N} A_{ijkl}^{(r)}D^r + D_{,m}\sum_{r=0}^{N} B_{ijklm}^{(r)}D^r\right\}\varepsilon_{kl} \tag{14}$$

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