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SPH-based simulation of granular collapse on an inclined bed

Hiroyuki Ikari*, Hitoshi Gotoh

Department of Civil and Earth Resources Engineering, Kyoto University, Kyoto, Japan

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ABSTRACT

The paper presents a numerical model for simulating a granular flow and its deposition on an inclined bed. A granular material is described as an elastic–plastic continuum and its constitutive law, namely Hooke's law, is discretized on the basis of the Smoothed Particle Hydrodynamics (SPH) method. In the equation of motion, however, the artificial viscosity, which is widely used in SPH, is not applied. The diffusive term derived from Hooke's law is introduced with a diffusion coefficient that varies depending on the stress and strain rate based on the Drucker–Prager yield function. The model is verified and validated through two numerical tests. It is shown that the basic elastic–perfectly plastic characteristics are reproduced with a simple shearing test. The effects of the diffusion coefficient and spatial resolution are investigated to show the validity of the model. In the simulation of the gravitational collapse of a granular column on an inclined bed, the performance of the model from the final deposition profile, the time history of the front position of the granular flow, the maximum runout distance, and the velocity profile are investigated for several cases of basal inclinations. The calculated results show good agreement with the experimental results.

1. Introduction

A granular flow is a phenomenon closely related to powder technology and geotechnical engineering. Especially, in geotechnical engineering, a granular flow can be treated as a simple model to predict the behavior of a landslide or debris flow. Hence, the deposition shape and runout distance are of considerable research interest.

A dam break, or the collapse of a granular column, is one of the simplest benchmark tests to investigate the behavior of a granular flow and verify the performance of a numerical model. Therefore, it has been carried out by many researchers using both experimental (e.g., [1–4]) and computational approaches. In a numerical simulation, a granular flow is generally described by two types of media: an aggregation of discrete elements and a continuum. The Discrete (or Distinct) Element Method (DEM) [5] is a widely used method for the former type. In this method, it is intuitively easy to explain the simulation results because each grain is expressed by a spherically shaped computational particle. A dam-break simulation by the DEM has been conducted (e.g., [6–8]). A treatment as a continuum was proposed by Savage and Hutter [9]. They showed that it is possible to describe the behavior of a finite mass of granular material by applying the equation of motion of a fluid, although a granular material is not a continuum. A 2D dam-break simulation has been carried out on the basis of the Navier–Stokes equation (e.g., [10–12]). In order to simulate the behavior of a granular flow, however, a viscosity coefficient that varies depending on the pressure and strain rate, like a Herschel–Bulkley fluid, is required (e.g., [12,13]). An elastic–plastic model can be also applied to the granular flow simulation (e.g., [14,15]). In the case where the granular material is regarded as soil and the prediction of a slip surface or stress field is needed, it is suitable to apply an elastic–plastic model.

The particle method, or the fully Lagrangian mesh-free method, such as the Smoothed Particle Hydrodynamics (SPH) method [16] and Moving Particle Semi-implicit (MPS) method [17], has been mainly applied to simulate fluid behavior such as a violent sloshing flow (e.g., [18,19]) and wave overtopping (e.g., [20,21]). A granular flow simulation using a particle method has also been carried out. For the particle method, a preliminary calculation of the random particle arrangement, which is needed in the DEM simulation, is not necessary, and it is easy to track the discrete behavior of a granular flow. For example, Minatti and Paris [22] and Liang and He [23] have simulated the collapse of a granular column by using a fluid-based model. Bui et al. [24] have succeeded in simulating large deformation of soil by an elastic–plastic model. However, it is difficult to reproduce the final deposition shape in a fluid-based model accurately owing to the pressure noise, which is an inevitable drawback of the standard particle method. In addition, an elastic–plastic

* Corresponding author. Tel.: +81 75 383 3312; fax: +81 75 383 3312.
E-mail address: ikari@particle.kuciv.kyoto-u.ac.jp (H. Ikari).

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model based on a particle method has not been validated for the inclined-bed case.

In the present paper, an SPH-based numerical model for simulating a granular flow is proposed and applied to the simulation of the collapse of a granular column on an inclined bed. The model treats a granular material as an elastic–plastic continuum; however, the artificial viscosity widely used in conventional SPH simulations is not included in the equation of motion. A diffusive term derived from the constitutive law is introduced with a diffusion coefficient that varies depending on the stress and strain rate, as in the models of Jop et al. [13] and Savage et al. [12]. The performance of the model will be demonstrated and compared to the experimental results by Hungr [3] and Lube et al. [4].

2. Numerical model

2.1. Governing equations

The governing equations are as follows:

$$\frac{D\rho}{Dt}+\rho\nabla\cdot\boldsymbol{u}=0 \tag{1}$$

$$\rho\frac{Du}{Dt}=\nabla\cdot\boldsymbol{\sigma}+\rho\boldsymbol{g} \tag{2}$$

where ρ is the density, $\boldsymbol{u}$ is the velocity vector, $\boldsymbol{\sigma}$ is the stress tensor, and $\boldsymbol{g}$ is the gravitational acceleration vector. The time derivative of $\boldsymbol{\sigma}$ is described with the Jaumann stress tensor as

$$\dot{\boldsymbol{\sigma}}=\dot{\boldsymbol{\omega}}\boldsymbol{\sigma}-\boldsymbol{\sigma}\dot{\boldsymbol{\omega}}+\lambda_e tr(\dot{\boldsymbol{\varepsilon}})\boldsymbol{I}+2\mu_e\dot{\boldsymbol{\varepsilon}}-\lambda_e tr(\dot{\boldsymbol{\varepsilon}}_p)\boldsymbol{I}-2\mu_e\dot{\boldsymbol{\varepsilon}}_p, \tag{3}$$

$$\dot{\boldsymbol{\varepsilon}}=\frac{1}{2}(\nabla\boldsymbol{u}+(\nabla\boldsymbol{u})^T), \tag{4}$$

$$\dot{\boldsymbol{\omega}}=\frac{1}{2}(\nabla\boldsymbol{u}-(\nabla\boldsymbol{u})^T), \tag{5}$$

in which $\dot{\boldsymbol{\varepsilon}}$ is the strain rate tensor, $\dot{\boldsymbol{\omega}}$ is the spin tensor, and $\boldsymbol{I}$ is the unit tensor. The Lamé parameters, λ_e and μ_e, are defined by the Young's modulus E and the Poisson's ratio ν, respectively, as follows:

$$\lambda_e=\frac{E\nu}{(1+\nu)(1-2\nu)},\quad \mu_e=\frac{E}{2(1+\nu)} \tag{6}$$

In the present model, soil, namely, dry granular material in this paper, is treated as an elastic–perfectly plastic material. The total strain rate $\dot{\boldsymbol{\varepsilon}}$ is divided into the elastic strain rate $\dot{\boldsymbol{\varepsilon}}_e$ and the plastic strain rate $\dot{\boldsymbol{\varepsilon}}_p$:

$$\dot{\boldsymbol{\varepsilon}}=\dot{\boldsymbol{\varepsilon}}_e+\dot{\boldsymbol{\varepsilon}}_p. \tag{7}$$

$\dot{\boldsymbol{\varepsilon}}_p$ can be evaluated on the basis of the plastic flow rule:

$$\dot{\boldsymbol{\varepsilon}}_p=\dot{\lambda}\frac{\partial g_p}{\partial\boldsymbol{\sigma}}, \tag{8}$$

in which g_p is the plastic potential function. $\dot{\lambda}$ is the rate of change of the plastic multiplier and can be obtained from the consistency condition, Hooke's law, and Eqs. (7) and (8) as follows [24]:

$$\dot{\lambda}=\frac{\lambda_e tr\left(\frac{\partial f_p}{\partial\sigma}\right)tr(\dot{\boldsymbol{\varepsilon}})+2\mu_e\frac{\partial f_p}{\partial\sigma}:\dot{\boldsymbol{\varepsilon}}}{\lambda_e tr\left(\frac{\partial f_p}{\partial\sigma}\right)tr\left(\frac{\partial g_p}{\partial\sigma}\right)+2\mu_e\frac{\partial f_p}{\partial\sigma}:\frac{\partial g_p}{\partial\sigma}}, \tag{9}$$

where f_p is the yield function. In the present model, the Drucker–Prager function is adopted as both the yield and plastic potential functions:

$$\left.\begin{aligned}f_p&=\sqrt{J_2}+\alpha_\phi I_1-\kappa\\ g_p&=\sqrt{J_2}+\alpha_\psi I_1\end{aligned}\right\} \tag{10}$$

$$\alpha_\phi=\frac{\tan\phi}{\sqrt{9+12\tan^2\phi}};\quad \alpha_\psi=\frac{\tan\psi}{\sqrt{9+12\tan^2\psi}};\quad \kappa=\frac{3c}{\sqrt{9+12\tan^2\phi}} \tag{11}$$

where $I_1=tr(\boldsymbol{\sigma})$ is the first invariant of the stress tensor, and J_2 is the second invariant of the deviatoric stress tensor. ψ is the internal friction angle, and c is the dilatancy angle, and c is the cohesion.

As for the problems of numerical errors such as tension cracking and a stress outside the yield surface, we address them using the same procedures as Bui et al. [24].

2.2. Diffusive term

Even if the term related to the divergence of the stress is discretized in an SPH formulation, we cannot carry out a stable calculation owing to the tension instability. Bui et al. adopted an artificial viscosity to avoid a numerical instability. The present model applies a diffusive term derived from the following procedures instead of the artificial viscosity.

The first term on the right hand side of Eq. (2) can be written with the time step k as follows:

$$\begin{aligned}\nabla\cdot\boldsymbol{\sigma}^{k+1}&=\nabla\cdot(\boldsymbol{\sigma}^k+\dot{\boldsymbol{\sigma}}\Delta t)\\&=\nabla\cdot\left\{\boldsymbol{\sigma}^k+\left(\dot{\boldsymbol{\omega}}^k\boldsymbol{\sigma}^k-\boldsymbol{\sigma}^k\dot{\boldsymbol{\omega}}^k+\lambda_e tr(\dot{\boldsymbol{\varepsilon}}^k)\boldsymbol{I}+2\mu_e\dot{\boldsymbol{\varepsilon}}^k\right.\right.\\&\qquad\left.\left.-\lambda_e tr\left(\dot{\boldsymbol{\varepsilon}}_p^k\right)\boldsymbol{I}-2\mu_e\dot{\boldsymbol{\varepsilon}}_p^k\right)\Delta t\right\}\end{aligned} \tag{12}$$

The divergence of the strain-rate tensor can be developed as follows:

$$\nabla\cdot\dot{\boldsymbol{\varepsilon}}=\frac{1}{2}\nabla\cdot tr(\dot{\boldsymbol{\varepsilon}})\boldsymbol{I}+\frac{1}{2}\nabla^2\boldsymbol{u}. \tag{13}$$

From Eqs. (12) and (13), we can obtain the following diffusive term.

$$\nabla\cdot\boldsymbol{\sigma}^{k+1}=\nabla\cdot\boldsymbol{\sigma}^*+\eta_0\nabla^2\boldsymbol{u}^k;\quad \eta_0=\mu_e\Delta t \tag{14}$$

$$\begin{aligned}\boldsymbol{\sigma}^*&=\boldsymbol{\sigma}^k+\left(\dot{\omega}^k\boldsymbol{\sigma}^k-\boldsymbol{\sigma}^k\dot{\boldsymbol{\omega}}^k+(\lambda_e+\mu_e)tr(\dot{\boldsymbol{\varepsilon}}^k)\boldsymbol{I}\right.\\&\qquad\left.-\lambda_e tr\left(\dot{\boldsymbol{\varepsilon}}_p^k\right)\boldsymbol{I}-2\mu_e\dot{\boldsymbol{\varepsilon}}_p^k\right)\Delta t\end{aligned} \tag{15}$$

We can conduct a stable simulation by applying the diffusive term in Eq. (14) as an additional viscosity to Eq. (2); however, we cannot accurately simulate the behavior of a granular material because the influence of the viscosity becomes too large in this diffusion coefficient. Hence, we adopt the following coefficient.

Savage et al. [12] derived a diffusion coefficient from the 2D noncohesive Mohr–Coulomb equation as follows:

$$\eta_{MC}=\frac{(I_1/2)\sin\phi}{2|\dot{\boldsymbol{\varepsilon}}|},\quad |\dot{\boldsymbol{\varepsilon}}|=\sqrt{\frac{1}{2}\dot{\boldsymbol{\varepsilon}}:\dot{\boldsymbol{\varepsilon}}} \tag{16}$$

This can be also described in the Drucker–Prager form as

$$\eta_{DP}=\frac{\alpha_\phi I_1}{2|\dot{\boldsymbol{\varepsilon}}|} \tag{17}$$

These diffusion coefficients vary depending on the stress and strain rate. The difference between Eqs. (16) and (17) is the parameter multiplying I_1. Fig. 1 shows the variation in these parameters in terms of the internal friction angle ϕ. η_{MC} is slightly larger than η_{DP} for the same friction angle and strain rate. In the present model, η_{DP} is applied instead of η_0; however, its range is limited as $\eta_{\min}\le\eta_{DP}\le\eta_{\max}$ in order to avoid zero or infinity. $\eta_{\min}$ and $\eta_{\max}$ are the minimum and maximum values, respectively.