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A three-dimensional yield-criterion-based flow model for avalanches

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ABSTRACT

In this paper, a flow model for avalanches based on the three-dimensional yield criterion is presented in an attempt to allow the relaxation of the assumption of lateral confinement pressure that is adopted in the traditional three-dimensional Savage–Hutter model (S–H model). One of the advantages of this model is that a simplified constitutive relationship for granular flow, which could reveal the internal mechanism of avalanches, is adopted. Additionally, another advantage is that the strength parameters used in the proposed model are readily available for natural materials. The flow properties of avalanches are influenced by the generalized friction coefficient, which is a parameter that can be assessed by introducing the three-dimensional yield criterion. By comparing the results obtained by numerical simulations using the model proposed in this paper and laboratory experiments, a reasonably good agreement can be reached with regard to the prediction of the moving process of avalanches.

1. Introduction

Avalanches, landslides, debris flows, and mudflows are dangerous natural hazards that always happen in mountainous regions, which could cause large numbers of casualties. Therefore, it is vital to develop reliable methods to simulate such types of mass movement. The flow process of avalanches can be modeled by either a discrete or a continuum approach. For a discrete approach, such as the distinct element method (DEM), the global properties of the granular material are studied based on an analysis of individual particles that obey the basic laws of motion [1–4]. Another method of modeling is to treat the flowing particles as an entity and adopt differential equations with integrated constitutive relations to describe the flow process. The governing equations for most methods in this category are derived by integrating the Navier–Stokes equations from the free surface to the base, and they can be distinguished based on the basal friction terms and the adopted constitutive relations.

Various dynamic models have been developed since the 1980s. In the early years, dam break and flood routing models with modified basal friction terms were used to model landslides and debris flows, e.g., Lang and Brown [5], Jeyapalan et al. [6], Takahashi [7,8], Voight and Sousa [9], and Shieh et al. [10]. These hydrodynamic methods adopted shallow water-type equations to simulate the debris flow process, i.e., the internal stress was assumed to satisfy the hydrostatic state. However, the basal friction terms differed in each of these models and most of them were empirically derived.

Generally, there are two types of mass flow in nature: quasi-static mass flow and dynamic mass flow. This classification is based on the ratio between the inertial grain collision stress and quasi-static Coulomb friction stress. Iverson et al. [11] introduced the Savage number to indicate the relative magnitude between the inertial grain collision stress and quasi-static Coulomb friction stress in granular flow. In the theory, the critical threshold value for whether a mass flow is quasi-static or dynamic mass is 0.1. It is indicated that most natural mass flows (e.g., avalanches, landslides, and debris flows) are quasi-static flows in which the Coulomb friction is dominant, i.e., the flows can be treated as frictional geotechnical materials. This frictional property has led to the wide adoption of the yield criterion to describe the state of stress in a moving mass. It has been proposed by several researchers such as Savage and Hutter [12], Iverson et al. [11], and Frenette et al. [13] that granular material begins to "flow" when its yielding condition is reached. It is also presumed in Bingham fluid theory that the mass will not flow until the stress reaches a certain value, i.e., the yield stress.

Savage and Hutter [12] assumed that the internal stresses in a moving mass obey the Mohr–Coulomb yield criterion in the direction of the dominating motion. Furthermore, the basal friction force was assumed to take the form of Coulomb friction, i.e., the shear stress was equal to the product of the normal stress and the friction coefficient. The derivations in this model were the active and passive earth pressure coefficients, which were used to describe

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the ratio of the horizontal to vertical stress. Hungr [14] presented different terms for the basal flow resistance force for different material rheologies (i.e., plastic flow, friction flow, Newtonian laminar flow, Bingham flow, Coulomb viscous flow, and Voellmy fluid), and for internal stress, the earth pressure coefficient was assigned based on the tangential strain prevailing at each mass block. In their model, Chen and Lee [15] used the earth pressure coefficient derived by Savage and Hutter [12] to describe the internal stress and introduced the principle of effective stress when evaluating the basal friction force.

To simulate avalanches flowing over three-dimensional terrain, Hutter et al. [16] extended the Savage–Hutter (S–H) theory by assuming that the stresses in the downslope direction dominate. Gray et al. [17] made a further improvement to the S–H theory for modeling avalanches flowing over irregular three-dimensional terrain. Other modifications of the S–H theory include two models proposed by Refs. [18,19]: one that relaxes the restrictions of small slope variations, and the other that takes erosion processes into account. Additionally, Fernández-Nieto et al. [20] proposed a new S–H type theory to simulate submarine avalanches and the associated generated tsunamis.

Although granular flow experiments show that the S–H theory depicts the flow of granular material well, some disadvantages remain. It is assumed in the S–H theory that the lateral confinement pressure is equivalent to a principal stress. However, this assumption is valid only under conditions when the “downhill” velocity and its variation are much bigger than the lateral velocity and its variation at each specific point in the flowing mass. Therefore, the S–H theory might have difficulty in theoretical verification when modeling avalanches flowing over complicated topographies. The S–H theory indicates a simple constitutive law such that the lateral stress depends only on the normal stress and the earth pressure coefficient. However, granular experiments [21] have revealed that the velocity and strain rate are factors that affect the internal stress of granular flow.

Jop et al. [22] proposed a constitutive relationship (i.e., $\mu(I)$-rheology) for granular material movement under conditions of shear. In their theory, internal stresses are closely connected with strain rates, and this proposed constitutive relationship has been used to interpret experimental results of dry granular flows [21]. However, if we apply this theory to the modeling of natural mass flows (i.e., landslides and debris flow), parameters such as grain size and the upper/lower bound friction angles are difficult to determine because the moving mass is a mixture of sediments with divergent grain sizes, clay, and water. Moreover, the method proposed by Jop et al. [22] is difficult to apply, in which a large number of experiments have to be conducted for the calibration of the parameters in the constitutive law. In this paper, we find the solution for this problem by importing the three-dimensional yield criterion into the simplified constitutive relationship. In this way, the physical parameters that are difficult to determine could be substituted by strength parameters, which are fundamental benchmarks used to evaluate the mechanical properties of soil. Furthermore, the use of the three-dimensional yield criterion might contribute to the relaxation of the assumption of the lateral confinement pressure, which would mean that the modified model could be applied to simulate avalanches flowing over complex topographies.

2. Governing equations

To describe a moving mass flowing on an inclined plane, a three-dimensional Cartesian coordinate system is established as follows: the z-coordinate follows the normal direction of the plane, and the x-coordinate and y-coordinate are two orthogonal axes parallel to the inclined plane (Fig. 1). Given that the angle between the x-coordinate and the horizontal plane is ζ and that the angle between the y-coordinate and the horizontal plane is θ, we can obtain the angle between the inclined plane and horizontal plane from $\beta = \mathrm{arc}\,cos(\sqrt{1-(\sin\zeta)^2-(\sin\theta)^2})$. If the elevation of the free surface and the bottom of the moving mass are $z = b(x, y, t)$ and $z = f(x, y, t)$, respectively, then the depth of the mass flow is $h(x, y, t) = f(x, y, t) - b(x, y, t)$.

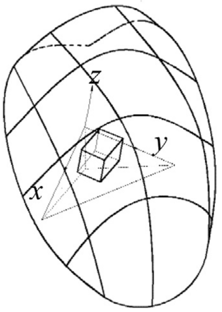

Fig. 1. Diagram of the basic element taken from the moving mass for analysis.

2.1. Simplified constitutive relations

If the lateral scale of the avalanche is much bigger than the flow depth, and the lateral velocity is much bigger than the vertical velocity, then the avalanche can be treated as “shallow” flow. Let us consider a cubic element of the entire flowing mass, clinging to the inclined plane, as the basic element for analysis (Fig. 1). As the gradient of vertical velocity is small in shallow granular flow, the shear stresses on the x–z plane and y–z plane can be neglected. The remaining stresses are shown in Fig. 2, where τ_{xz} and τ_{yz} are shear stresses generated by basal friction and p_{xx}, p_{yy} and p_{zz} are normal stresses in three directions.

Jop et al. [22] proposed a phenomenological frictional viscoplastic constitutive law, which is usually referred to as $\mu(I)$-rheology. Based on dimensional analysis, $\mu(I)$-rheology is proposed by analyzing large amount of experimental and numerical data sets. It reveals a common framework describing a wide range of behavior of granular materials [23]. The rheology is different from the description developed in other viscoplastic materials in two main aspects: (1) the effective viscosity depends on the pressure, not just on the shear rate; (2) the flow threshold is not a material constant but also depends on the pressure [24]. And it is indicated that the internal stresses are closely related to the isotropic pressure, strain rate, and second invariant of the strain rate [22].

We introduce the generalized friction coefficient υ into the constitutive relation to describe the property of the moving mass; the definition of the generalized friction coefficient will be discussed in the following. While for the basal friction, it is assumed to be Coulomb friction and μ_{bx} and μ_{by} denote the basal frictional

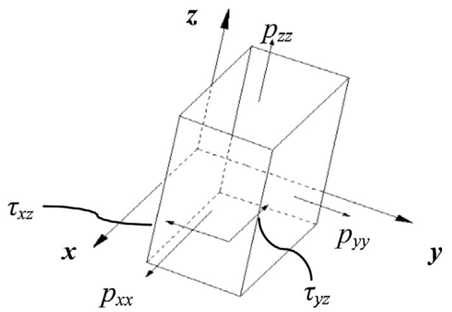

Fig. 2. Diagram of stress analysis for the basic element.