Mechanics Research Communications 73 (2016) 39–46

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Modelling a micro-cantilever vibrating in vacuum, gas or liquid under thermal base excitation

Mojtaba Komeili*, Carlo Menon

MENRVA Research Group, School of Engineering Science, Simon Fraser University, Metro Vancouver, Canada

ARTICLE INFO

Article history:
Received 22 July 2015
Received in revised form 6 January 2016
Accepted 26 January 2016
Available online 3 February 2016

Keywords:
Micro-electro-mechanical systems (MEMS)
Micro-cantilever beam
Harmonic response
Thermal excitation
Fluid dynamics

ABSTRACT

The dynamic behaviour of a micro-cantilever that is transversely excited at its base is investigated in this paper. The base actuation is provided by thermal cycles via taking the advantage of thermal expansion. The Euler–Bernoulli equation along with corresponding boundary conditions is used to model the continuous cantilever beam. The resultant boundary value problem takes into account the thermal expansion and stiffness of the actuator at the base as well as the effect of the surrounding gas or liquid. A closed-form analytical model is developed to compute natural frequencies, mode shapes, and harmonic response of the vibrating cantilever, in addition to an integral function for quality factor. The model is validated via a finite element (FE) analysis using ANSYS commercial package. This validation shows that the proposed model can properly predict the cantilever's vibrating behaviour.

1. Introduction

The introduction of micro-electro-mechanical systems (MEMS) in the last few decades has accelerated the advancement of technology in various areas and fields. MEMS technology has wide applications in aerospace, defence, automotive, consumer electronic products, biomedical devices, and even emerging technology sectors. A comprehensive discussion on various applications has been presented by Maluf and Williams [1]. In general, MEMS devices are used for sensing or/and actuation. Amongst different architectures used for this purpose at micro-scale, micro-beams and particularly micro-cantilever beams are of high interest, primarily due to their simple design [2]. Micro-cantilevers are used in probe of scanning force microscopy and scanning tunnel microscopy [3], mass sensing [4], wafer probes [5], energy harvesters and micro-generators [6], bio-sensors [7,8], frequency doublers [9] and many other purposes [1].

It should be noted that in most of the above-mentioned applications the micro-cantilevers vibrate at high frequencies through excitation applied via actuators. There are various techniques to actuate MEMS devices to vibrate at high frequencies. Some of the most common approaches include: thermal, electrostatic, electromagnetic, and piezoelectric actuation [2]. Thermal actuation has the advantage of producing relatively large force and displacement. Moreover, it has been proven that in practice thermal cycles can be easily applied in order of a few hundred MHz [10]. This allows them to be effectively implemented in the resonant MEMS devices [11]. In addition, thermal actuators are generally easier to manufacture due to the fact that the main principle behind this type of actuation is thermal expansion and a wide range of materials can be used. Temperature rise can be simply provided through an electric current and cooling cycle is facilitated by conduction to the bases and substrate, convection to the ambient fluid (gas) and/or radiation. On the other hand, the amount of power consumption in the thermal actuators is relatively high and the hazard of overheating is significant. Thus, thorough analysis and design is necessary for the thermally actuated devices. In general, in most cases thermally actuated MEMS are excited in their resonance frequency (or frequencies close to it) in order to increase the performance [12]. Therefore, the analysis of their free vibration in order to detect the resonance/natural frequencies and their behaviour at these frequencies are extremely important.

This paper investigates a simple yet effective actuation technique to vibrate a generic micro-cantilever through vertical excitation of its base. This is achieved by an element, the actuator, which expands when heated. An analytical formulation for finding the natural frequency of the device and its response to the harmonic heating cycle is studied. The characteristic equation and closed form formulas representing the displacement and stress along the cantilever are presented.

There have been numerous studies in the past for vibration analysis of micro-cantilevers [13–15]. Wu and Chang [16] modelled

* Corresponding author. Tel.: +1 778 782 6860; fax: +1 778 782 4951.
E-mail address: carlo_menon@sfu.ca (M. Komeili).

http://dx.doi.org/10.1016/j.mechrescom.2016.01.009
0093-6413/

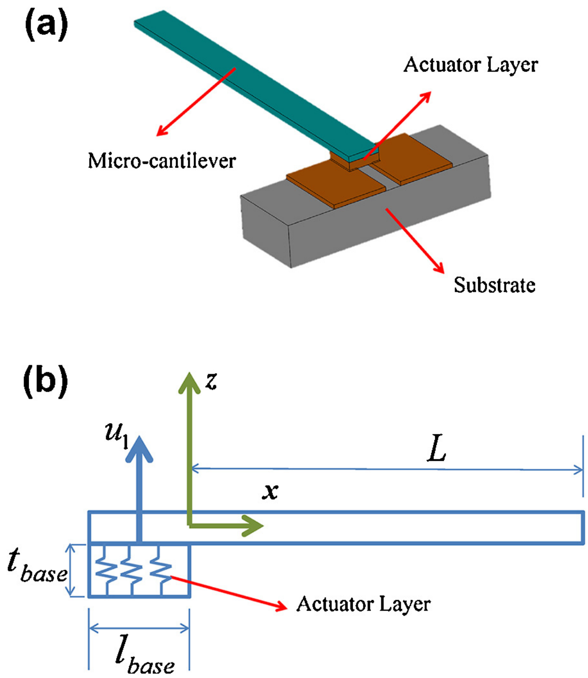

Fig. 1. (a) Geometry of the cantilever and actuator in a MEMS device; (b) the schematic of the mechanical model used for driving the equations of motion and boundary conditions.

vibrations of a micro-cantilever that was thermally excited via the bimorph effect. As an attempt to apply excitation at the base of the cantilever, Bouwstra et al. [17] manufactured and tested a micro-cantilever actuated via base rotation. Later on, Syms [18] took advantage of base rotation using cold and hot arms for rotation of beam. Similarly, Heinrich et al. [19] tackled the case of a cantilever vibrating laterally (in-plane) using a simplified approach via applying harmonic rotations at its base. However, besides the slight difference between their methods of excitation (base rotation) and current approach (vertical motion at base), the effect of temperature change on the vibration was not considered directly and was transformed into its equivalent rotations.

This paper presents the analytical model of a cantilever Euler–Bernoulli beam mounted on an elastic base (actuator) and surrounded by an ambient fluid. Electro-thermal actuation at the base is taken into account by harmonic temperature change in the actuator. As a result, the expansion and contraction cycles in the elastic base results in vibrations of the beam. The boundary value problem is solved analytically to find the characteristic equation, natural frequency and mode shapes. Results are compared to result of modal analysis using ANSYS^{TM}. Similarly, the harmonic response of the system is extracted as a closed form formula. Validation of the harmonic response is made with an analogous case in ANSYS, which applies displacement-controlled excitation at the base. It has been shown that, applying the resultant displacement from a thermal expansion can be used as a coherent approximation to study thermal actuation [19]. Presence of ambient fluid induces hydrodynamic forces and damping, therefore the quality factor for the system is formulated as well.

2. Micro cantilever

Fig. 1(a) shows a schematic design of the studied thermally actuated cantilever. The actuator, i.e. the expanding element, is placed underneath the beam. In this figure, the actuator is connected to conductors that provide the power to heat up the actuator via Joule effect. Fig. 1(b) shows the schematic of the mechanical model used for studying the behaviour of this system. As it can be seen in the figure, the actuator is assumed to be a block of solid material that contributes to the vibration response through its stiffness, which can simply be calculated via:

$$k_{base} = \frac{E_{base}A_{base}}{t_{base}} \tag{1}$$

where k_{base} is the stiffness, E_{base} is the Young modulus of the material used in the base, A_{base} is the cross section area of actuator (simply calculated by l_{base} times out-of-plane thickness, w), and t_{base} is the thickness of actuation layer.

3. Boundary value problem

3.1. Continuous beam equations

The equation of motion for the Euler–Bernoulli beam under small deformation can be derived as following [20]:

$$\mathrm{EI}\frac{\partial^4 u(x,t)}{\partial^4 x} + m\frac{\partial u^2(x,t)}{\partial t^2} = q(x,t) \tag{2}$$

where $u(x,t)$ is the total beam deflection. E is the stiffness of the beam material and I is the second moment of inertia relative to the neutral axis of the beam; m represents the weight per unit length of the beam; $q(x,t)$ is the distributed load on the beam (in this case resulted from fluid forces along the beam); and x and t are the spatial coordinate along the length of the beam and time, respectively. In general, the above equation can be solved using the boundary (and initial) conditions that are imposed on the system. Defining F_1 and u_1 as the force and displacement at the top of actuator, respectively, the following boundary conditions can be identified for the micro-cantilever depicted in Fig. 1(b):

$$\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=0} = 0 \tag{3a}$$

$$\mathrm{EI}\left.\frac{\partial^3 u(x,t)}{\partial x^3}\right|_{x=0} = -F_1 \tag{3b}$$

$$\left.\frac{\partial^2 u(x,t)}{\partial x^2}\right|_{x=L} = 0 \tag{3c}$$

$$\left.\frac{\partial^3 u(x,t)}{\partial x^3}\right|_{x=L} = 0 \tag{3d}$$

Using the thermo-elastic formulation for a one-dimensional element (i.e., actuator), and taking into account thermal expansion:

$$F_1 = u_1 k_{base} - \alpha t_{base} k_{base} \Delta\theta(t) \tag{4a}$$

$$u_1 = u(0,t) = C_2 + C_4 \tag{4b}$$

where α is the linear expansion coefficient of the actuator material.

Note that $\Delta\theta(t)$ in this formula is the source for the thermal actuation. Indeed, the thermal expansion in the actuator results in a combination of force and displacement (F_1 and u_1), which leads to the vibration of the micro-cantilever. In this study only the free vibration and harmonic response of the beam is considered, thus $\Delta\theta(t)=0$ and $\Delta\theta(t) = \overline{\Delta\theta}e^{i\omega t}$ are considered respectively, where $i = \sqrt{-1}$, $\omega = 2\pi f$, and f is the frequency of harmonic vibration.

3.2. Hydrodynamic force

For solving the partial differential equation (PDE) presented in Eq. (2), the method of separation of variable [21] can be used by replacing $u(x,t) = w(x)g(t)$. Considering the fact that the system is under harmonic loading and field variables behave harmonically (linear and small deformation assumption), the time-dependency