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Modeling of thermo-electro-magnetic-elastic waves in a transversely isotropic circular fiber

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ABSTRACT

In this article, wave propagation in a transversely isotropic thermo-electro-magneto-elastic circular fiber is studied. The equation of motion are formulated using the three dimensional theory of thermo-electro-magneto elasticity. The displacement, electric, magnetic and thermal potentials are introduced to uncouple the equations of motion. The frequency equations for both longitudinal and flexural modes of vibration are derived and the numerical results are plotted in the form of dispersion curves. The frequency equation of an electro-magneto-elastic circular fiber is derived by omitting the thermal field and the dispersion behavior of thermo-electro-magneto elastic and electro-magneto-elastic circular fiber is discussed in the form of comparison graphs.

1. Introduction

The vibration responses of thermo-electro-magneto-elastic materials have been developed extraordinarily due to its applications in material science, engineering and solid-state physics. The practical applications include magnetic storage elements, geophysical physics, magnetic structural elements, plasma physics, transducers, phase in vectors, sensors, wave guides, etc. The thermal stresses in the elastic materials are used in thermal power plants, chemical pipes, pressure vessel, aerospace and metallurgy.

Dai and Wang [9] analyzed the electromagnetic elastic responses of piezoelectric hollow cylinders under a sudden mechanical load and electric potential. Later the same authors Dai and Wang [10] presented an analytical method for stress wave propagation of spherically symmetric motion in laminated piezoelectric spherical shells subjected to thermal shock and electric excitation loads by applying Hankel transforms and Laplace transforms. Dong and Wang [11] investigated the effect of large deformation and rotary inertia on wave propagation in long, piezoelectric cylindrically laminated shells in thermal environment. Aouadi [3] modeled the equation of generalized thermo-piezoelectricity in an isotropic elastic medium with temperature dependent mechanical properties. Further the author adopted a state-space approach for the solution of one-dimensional problems. Saadatfar and Razavi [26] presented an analytical solution for the axisymmetric problem of a piezoelectric hollow cylinder with thermal gradient. Based on Hamilton's principle, Maxwell equation and first-order shear deformation theory Sheng and Wang [27] investigated the thermoelastic vibration and buckling properties of functionally graded piezoelectric cylindrical shell. Chang and Wang [7] studied the effect of thermal shock on a finite piezoelectric cylinder with embedded penny-shaped crack by applying the theory of linear electro-elasticity.

Huang et al. [15] obtained an exact dispersion relation of a shear waves propagating in a piezoelectric and piezomagnetic phase with imperfect bonded interface. Pang et al. [18] studied the dynamic behavior of wave propagation in piezoelectric and piezomagnetic layered periodic composites. Du and Xian [12] investigated the shear horizontal surface acoustic wave propagation in a cylindrically layered magneto-electro-elastic structure by means of Laplace transform and Bessel equation. Dai et al. [8] analyzed the electromagnetoelastic behavior of a functionally graded piezoelectric solid cylinder and sphere subjected to uniform magnetic field, external pressure and electric loading. Meeker and Meitzler [16] investigated the wave propagation in elongated cylinders and plates. Two parts study by Mirksy [17] devoted the problem of longitudinal wave propagation in transversely isotropic circular cylinders using an idea based on potential functions. Tiersten [28] investigated vibrations of linear piezoelectric plates.

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Parton and Kudryavtsev [19] presented the electroelastic governing equations for piezoelectric materials. Using Fourier expansion collocation method Paul and Venkatesan [21] studied the wave propagation in a piezoelectric ceramic cylinder of arbitrary cross-section with circular cylindrical cavity. Rajapakse and Zhou [25] analyzed Fourier integral transform. Wang [29] studied the axisymmetric wave propagation in cylinder coated with the piezoelectric layer. Ebenezer and Ramesh [13] analyzed the axially polarized piezoelectric cylinders with arbitrary boundary conditions on the flat surfaces using Bessel function. Berg et al. [4] discussed the dynamic characteristics of piezoelectric cylindrical shell. Botta and Cerri [5] studied the wave propagation in Reissner–Mindlin piezoelectric coupled cylinder with non-constant electric field through the thickness. The wave motion in elastic solids has been studied in detail by Graff [14] and Achenbach [2]. Ponnusamy [22,23] studied the frequency responses of generalized thermo elastic solid cylinder of arbitrary cross-section and plate of polygonal cross-section using the Fourier expansion collocation method. Later the same author Ponnusamy [24] discussed the wave propagation of a piezoelectric solid bar of circular cross-section immersed in fluid using secant method.

The present study is aimed to investigate the frequency responses of transversely isotropic thermo-electro-magneto-elastic circular fiber. The frequency equations are obtained for thermo-electro-magneto-elastic circular fiber and in particular case the frequency equations are also obtained for electro-magneto-elastic circular fiber by omitting the thermal field in the constitutive equations of motion. The non-dimensional wave numbers are calculated for both longitudinal and flexural modes of vibration and are plotted in the form of dispersion curves. Further comparison graphs are drawn to discuss the dispersion behavior of thermo-electro-magneto-elastic and electro-magneto-elastic circular fiber.

2. Formulation of the problem

We consider a homogenous electro-magneto-thermo elastic circular fiber of infinite length. The complete equations governing the behavior electro-magneto elastic fiber have been considered from Buchanan [6]. In cylindrical coordinates (r, θ, z) the equations of motion in the absence of body force are

$$\begin{aligned}
&\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho u_{r,tt} \\
&\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{z\theta,z} + 2r^{-1}\sigma_{r\theta} = \rho u_{\theta,tt} \\
&\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{rz} = \rho u_{z,tt}.
\end{aligned} \tag{1}$$

The heat conduction equation is

$$K_1\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta}\right) + K_3 T_{,zz} - \rho c_v\left(\dot{T} + t_0\ddot{T}\right) = T_0\left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right)\left[\beta_1(S_{rr} + S_{\theta\theta}) + \beta_3 S_{zz} - p_3\Phi_{,z} - p_4\Psi_{,z}\right]. \tag{2}$$

The electric displacements D_r, D_θ and D_z satisfy the Gaussian equation is

$$\frac{1}{r}\frac{\partial}{\partial r}(rD_r) + \frac{1}{r}\frac{\partial D_\theta}{\partial \theta} + \frac{\partial D_z}{\partial z} = 0. \tag{3}$$

The magnetic displacement B_r, B_θ and B_z satisfy the equation is

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_z}{\partial z} = 0. \tag{4}$$

The governing equations that relate the thermo-electro-magneto-elastic circular fiber are

$$\begin{aligned}
&\sigma_{rr} = c_{11}S_{rr} + c_{12}S_{\theta\theta} + c_{13}S_{zz} - \beta_1 T - e_{31}E_z - q_{31}H_z, \\
&\sigma_{\theta\theta} = c_{12}S_{rr} + c_{11}S_{\theta\theta} + c_{13}S_{zz} - \beta_1 T - e_{31}E_z - q_{31}H_z, \\
&\sigma_{zz} = c_{13}S_{rr} + c_{13}S_{\theta\theta} + c_{13}S_{zz} - \beta_3 T - e_{33}E_z - q_{33}H_z, \\
&\sigma_{r\theta} = 2c_{66}S_{r\theta}, \\
&\sigma_{\theta z} = 2c_{44}S_{\theta z} - e_{15}E_\theta - q_{15}H_\theta, \\
&\sigma_{rz} = 2c_{44}S_{rz} - e_{15}E_r - q_{15}H_r,
\end{aligned} \tag{5}$$

$$\begin{aligned}
&D_r = 2e_{15}S_{rz} + \varepsilon_{11}E_r + m_{11}H_r, \\
&D_\theta = 2e_{15}S_{\theta z} + \varepsilon_{11}E_\theta + m_{11}H_\theta, \\
&D_z = e_{31}(S_{rr} + S_{\theta\theta}) + e_{33}S_{zz} + \varepsilon_{33}E_z + m_{33}H_z + p_3 T,
\end{aligned} \tag{6}$$

and

$$\begin{aligned}
&B_r = 2q_{15}S_{rz} + m_{11}E_r + \mu_{11}H_r, \\
&B_\theta = 2q_{15}S_{\theta z} + m_{11}E_\theta + \mu_{11}H_\theta, \\
&B_z = q_{31}(S_{rr} + S_{\theta\theta}) + q_{33}S_{zz} + m_{33}E_z + \mu_{33}H_z + p_4 T,
\end{aligned} \tag{7}$$

where u_r, u_θ and u_z are respectively the radial, tangential and axial displacement components, which are defined through the cylindrical coordinates r, θ and z. $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{r\theta}, \sigma_{\theta z}, \sigma_{rz}$ are the stress components, $S_{rr}, S_{\theta\theta}, S_{zz}, S_{r\theta}, S_{\theta z}, S_{rz}$ are the strain components, T is the temperature change about the equilibrium temperature T_0, c_{11}, c_{12}, c_{13}, c_{33}, c_{44} and $c_{66} = (c_{11} - c_{12})/2$ are the five elastic constants, β_1, β_3 and K_1, K_3 respectively denotes thermal expansion coefficients and thermal conductivities along and perpendicular to the symmetry, e_{31}, e_{15}, e_{33} are the piezoelectric constants, ε_{11}, ε_{33} are the dielectric constants, q_{31}, q_{15}, q_{33} are the piezomagnetic material coefficients, m_{11}, m_{33}