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Nonlinear stability and dynamics of composite skew plates under nonuniform loadings using differential quadrature method

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ABSTRACT

The buckling, postbuckling and postbuckled vibration behaviour of composite skew plates subjected to nonuniform inplane loadings are presented here. The skew plate is modelled using first order shear deformation theory accounting for von-Kármán geometric nonlinearity and initial geometric imperfections. The different types of nonuniform loads that have been considered in this study are concentrated load, partial load and parabolic load. The explicit analytical expressions for prebuckling stress distributions within composite skew plate subjected to three different types of nonuniform inplane loadings are developed by solving plane elasticity problem using Airy's stress function approach. It is observed that the inplane normal stress distributions within the skew plate due to above nonuniform loadings do not become uniform even at mid-section. The generalized differential quadrature (GDQ) method is used to solve the nonlinear governing partial differential equations. It is observed that the postbuckled load carrying capacity of skew plate under concentrated loading is the lowest compared to other nonuniform and uniform loadings.

1. Introduction

The buckling, postbuckling and postbuckled vibration analysis of composite skew plates under uniform loading is studied by researchers using both classical and finite element methods. Composite skew plates are important structural elements in modern engineering structures such as aircrafts, space vehicles, missiles and many complex structures. Quite often, these plates are subjected to different types of nonuniform mechanical inplane loadings. To understand the performance of these structural elements under different nonuniform inplane loadings, buckling, postbuckling and postbuckled vibration results of layered composite skew plates are reported here. Expressing displacement function as double Fourier sine series in Rayleigh Ritz method, Durvasula [1] reported buckling load of simply supported isotropic skew plate under uniform inplane loading. The rate of convergence became poor with the increased skew angle due to difficulty of satisfying the natural boundary conditions and hence reported higher buckling loads. Edwardes and Kabaila [2] identified the presence of stress singularity at the obtuse corners and relaxed the boundary condition of zero slopes at the obtuse corners to obtain better results and also adopted mesh grading technique at the obtuse corners for better convergence within the framework of finite element method (FEM). Using Pb-2 Ritz function (the product of two dimensional polynomial functions and a basic function), Wang et al. [3] reported the critical buckling load of thin isotropic skew plates via Ritz method. The critical buckling loads were presented for two types of corner conditions (zero moment and zero slopes) that can exist for simply supported boundary conditions. The critical buckling load of skew plate for zero moment boundary conditions at corner was lower than zero slopes boundary conditions at corner. Extending the above method, Kitipornchai et al. [4] obtained the buckling load of thick skew plates under uniform inplane loading. Higher buckling load was reported for simply supported boundary conditions due to zero slope boundary conditions at the obtuse corners. Two finite element models based on first order shear deformation theory (FSDT) and higher order shear deformation theory (HSDT) were developed by Babu and Kant [5,6] to study the effect of shear deformation on the mechanical and thermal buckling loads of laminated and sandwich skew plates under uniform inplane loading. Karami et al. [7] studied the static, free vibration and buckling behaviour of laminated composite skew and trapezoidal thin plates using differential quadrature method under uniform inplane loading.

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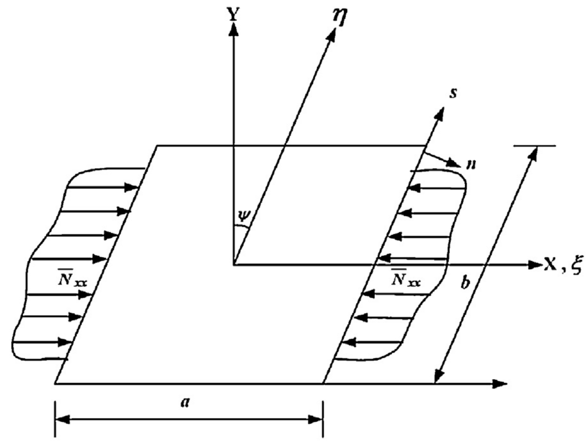

Fig. 1. Geometry and nonuniform loading for skew plate.

Singha et al. [8] studied the buckling and postbuckling behaviour of shear deformable composite skew plates subjected to combined uniaxial compression and uniform temperature rise through the thickness using finite element method. A maximum strength criterion was used to determine the failure of laminates. This was compared with the failure temperature calculated using Tsai-Hill and Tsai-Wu criteria. Daripa and Singha [9] studied the effect of corner stress on the buckling and postbuckling response of composite skew plates using the shear deformable finite element method. Tahmasebi and Shanmugam [10] and Hu and Tzeng [11] used ABAQUS (finite element software package) to obtain the critical buckling load of skew laminated plates with and without openings under uniform inplane loading. Jaberzadeh and Boroomand [12] studied the inelastic buckling of skew and rhombic (thin) thickness-tapered plates with and without intermediate supports using element free Galerkin's method. In this method, shape functions were constructed using the moving least square method. The postbuckled vibration characteristic of anisotropic plate is studied using finite element method [13] and Galerkin method [14] based on first order shear deformation theory (FSDT). Singha et al. [15] studied the exchange of vibration modes in the thermally post-buckled skew plates using FEM. Xia and Shen [16] studied the small- and large-amplitude vibrations of thermally post-buckled functionally graded plates with piezoelectric actuators subjected to uniform and nonuniform temperature fields using improved perturbation technique.

The objective of this paper is to compute, postbuckling and postbuckled vibration behaviour of composite skew plates subjected to three different types of inplane loadings distributions – concentrated, partial and parabolic. The analytical expressions for stress functions for composite skew plate due to these nonuniform loadings are derived by solving the plane elasticity problem by the Airy's stress function approach for the first time. Subsequently, using these inplane stress distributions within the plate, the governing equations for buckling, postbuckling and postbuckled vibration analysis of layered composite skew plates are formulated in terms of displacement (u,v,w) and rotation (ϕ_x, ϕ_y) variables. Using GDQ, the resulting governing partial differential equations and boundary conditions are reduced into a set of nonlinear algebraic equations for postbuckling analysis and nonlinear ordinary differential equation for postbuckled vibration analysis. GDQ method is chosen because of its ability to provide accurate results with relatively lesser computational effort than other numerical methods [17–19]. The nonuniform (Chebyshev-Gauss-Lobatto) grid points are considered for better accuracy because more grid points are required at the boundary [20] to accurately capture the stress gradients than at the inner region. Lagrange interpolated polynomials are employed as test function [21,22] for calculating the weighting coefficients. The nonlinear algebraic equations are solved using modified Newton–Raphson method to trace the postbuckling equilibrium paths. The frequencies of postbuckled skew plate are reported by solving the associated eigenvalue problem for different postbuckled equilibrium states. The influences of skew angles, initial imperfections, boundary conditions and nonuniform inplane loadings on the buckling, postbuckling and postbuckled vibration behaviour of the composite skew plate are investigated.

2. Formulation

2.1. Governing equation

Consider a laminated composite skew plate of length a, breadth b and thickness h composed of N orthotropic layers of uniform thickness and subjected to nonuniform loading as shown in Fig. 1. Let the middle plane of the skew plate coincide with $\xi-\eta$ axes of the skew co-ordinate ($\xi-\eta-z$) system. In the present formulation, the skew plate is modelled using first order shear deformation theory incorporating von Kármán geometric nonlinearity and (geometric) imperfections. The equation of motion of layered composite skew plate subjected to nonuniform inplane loading in the cartesian co-ordinate system are obtained in terms of force, moment and shear resultants respectively via Hamilton principle and are,

$$
\begin{aligned}
&\delta u : \hat{N}_{xx,x} + \hat{N}_{xy,y} = \rho h u^o_{,tt} \\
&\delta v : \hat{N}_{xy,x} + \hat{N}_{yy,y} = \rho h v^o_{,tt} \\
&\delta w : Q_{xx,x} + Q_{yy,y} + \hat{N}_{xx} w,_{xx} + 2\hat{N}_{xy} w,_{xy} + \hat{N}_{yy} w,_{yy} = \rho h w^o_{,tt} \\
&\delta\phi_x : M_{xx,x} + M_{xy,y} - Q_{xx} = (\rho h^3/12)\phi^o_{x,tt} \\
&\delta\phi_y : M_{xy,x} + M_{yy,y} - Q_{yy} = (\rho h^3/12)\phi^o_{y,tt}
\end{aligned}
\tag{1a-e}
$$