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Decay rates in nano tubes with consideration of surface elasticity

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ABSTRACT

In the present paper we examine the Saint-Venant end effect in the nano tubes via a continuum mechanics with consideration of surface elasticity. The Saint-Venant end effect is quantitatively described by the decay rate. By analytically solving an axial-symmetric torsion in a circular cross-section tube configuration, we demonstrate that the decay rate decreases as the radius of the nano wire/tube decreases with consideration of the surface effect.

1. Introduction

When people study the deformation of the nano wires and nano tubes, continuum mechanics has been one of the powerful options [1]. It has also been noticed that on top of the classical continuum mechanics theory for the bulk materials, the surface elasticity must be included in the analysis as the surface to bulk ratio is considered to be high in nano wire or nano tube configurations. The surface mechanics theory addressing the surface effect was proposed by Gurtin and Murdoch [2], and Gurtin et al. [3]. We will adopt the spirit of their theory of the surface elasticity in the present study on the end-effect of the nano wires and nano tubes.

Regarding the end effect, we would like only trace back the research activities in the recent decades (e.g., a review article by Horgan [4]). The end effect was named after Saint-Venant with great conceptual importance in the classical mechanics of materials. For a slender bar, Saint-Venant principle stated that the balanced end forces can only affect the end region which is about the distance of the characteristic dimension of the cross section of the bar. A rigorous mathematical theory has been formulated after 1980s by using the perturbation asymptotic expansion concept. The Saint-Venant solution is the outer expansion, and the end effect is the inner expansion in the perturbation theory. The contributions by Gregory and Wan [5], Fan and Widera [6] are some representative works along this line, in which the end effect was quantitatively estimated via the concept of the decay rate. More recently, the decay rates concept was introduced into non-classical materials, such as piezoelectric materials (e.g., Fan [7] and Pan et al. [8]).

In the present study, we consider an axial symmetric torsion end-load effect for its simplicity in mathematics and availability of analytical solution. Although there are other types of the decay rates associated with other types of loadings (such as tension, bending and shear), they are the same in concept. Our main objective here is to estimate the decay rate with consideration of the surface effect as the surface to bulk ratio increases drastically for the nano wires/tubes. An important parameter introduced in our derivation is a surface/size factor, $S_\mu = \mu_s/\mu b$, which will be defined in details in the following sections.

2. The decay solution

Let us consider an axial symmetric torsion problem in a circular cylinder as shown in Fig. 1(a).

The non-zero stress and displacement components are

$$\sigma_{r\theta} = \sigma_{r\theta}(r, z), \quad \sigma_{z\theta} = \sigma_{z\theta}(r, z), \quad \text{and } u_\theta = u_\theta(r, z). \tag{1}$$

The equilibrium, Hooke's law and deformation equations, after considering the axial symmetric torsion condition, are given as

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{\partial \sigma_{z\theta}}{\partial z} + \frac{2\sigma_{r\theta}}{r} = 0 \tag{2}$$

$$\sigma_{r\theta} = 2\mu\varepsilon_{r\theta} \quad \sigma_{z\theta} = 2\mu\varepsilon_{z\theta} \tag{3}$$

$$2\varepsilon_{r\theta} = \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}, \quad 2\varepsilon_{z\theta} = \frac{\partial u_\theta}{\partial z} \tag{4}$$

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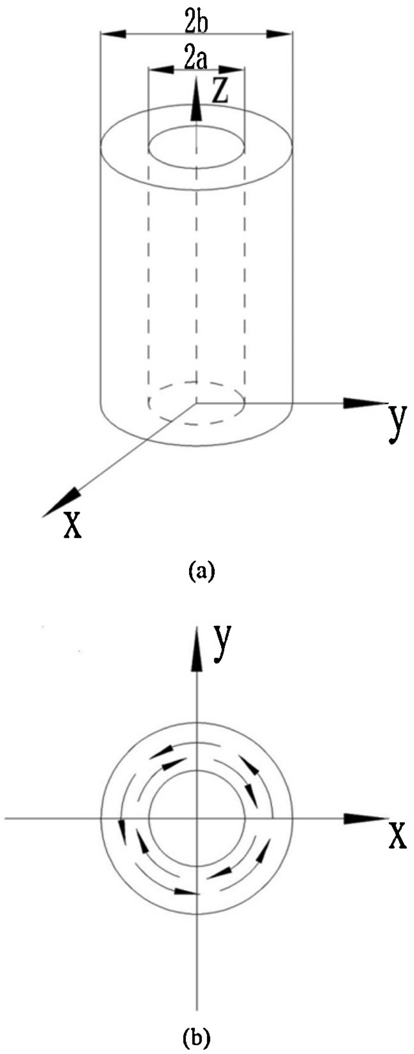

Fig. 1. (a) Geometric dimension of the tube. (b) Self-equilibrium torsion load at the end $Z = 0$.

where μ is the shear modulus.

The boundary conditions for solving the partial differential Eqs. (2)–(4) are given at the ends and circumferential surfaces of the tube. The end cross sections are prescribed by a self-balanced torsional loading, i.e.

$$\int_{S_0} r\sigma_{z\theta}(r,0)\mathrm{d}r = 0 \tag{5}$$

The stress pattern described by Eq. (5) is schematically shown in Fig. 1(b). The circumferential surfaces of the solid are covered by two-dimensional surface membranes which are described the surface elasticity [2,3].

Instead of using general formulation of the bulk-surface interaction, hereby, we would like to focus on our simplified geometric and loading configuration. Referring to Fig. 2 and the condition of axial symmetric torsion configuration, we have the equilibrium equation for the surface membrane element as

$$\frac{\partial \sum_{z\theta}}{\partial z} + \sigma^{+}_{r\theta} - \sigma^{-}_{r\theta} = 0 \tag{6}$$

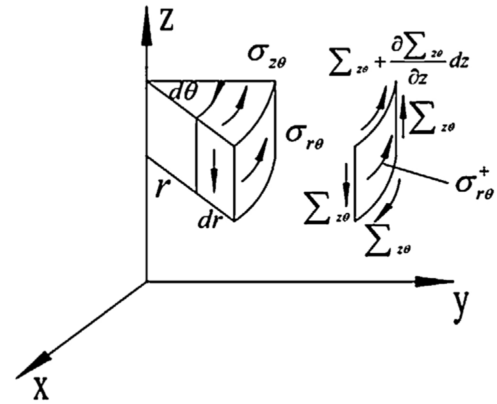

Fig. 2. The bulk element and the membrane element.

where $\Sigma_{z\theta}$ is the surface stress acting on the membrane element with a dimension of N/m. The surface membrane material obeys the corresponding Hooke's law

$$\sum\nolimits_{z\theta} = 2\mu_s \varepsilon_{z\theta} \tag{7}$$

where μ_s is the shear modulus of the membrane with dimension of N/m. Also, implied in Eq. (7), we used the assumption of the displacement continuity between the bulk element and the membrane element. Thus, the strain components are the same in the bulk and the membrane element, and are related to the displacement via Eq. (4).

Before we proceed to the solution of the boundary value problem defined by Eqs. (2)–(7), a set of non-dimensional equations is needed to unveil the size and surface effect. We introduce a set of dimensionless coordinate and displacement as follows:

$$\xi = \frac{r}{b}, \quad \eta = \frac{z}{b}, \quad U_\theta(\xi,\eta) = \frac{u_\theta(r,z)}{b}. \tag{8}$$

where b is the outer radius of the wire/tube shown in Fig. 1(a). The dimensionless coordinates ranges are given by $R \le \xi \le 1$ and $0 \le \eta < \infty$, where the dimensionless inner radius is $R = a/b$.

With these dimensionless coordinates the displacement, Eqs. (2)–(4) are combined and rewritten as:

$$\frac{\partial \sigma_{r\theta}}{\partial \xi} + \frac{\partial \sigma_{z\theta}}{\partial \eta} + \frac{2\sigma_{r\theta}}{\xi} = 0 \tag{9}$$

$$\sigma_{r\theta} = \mu\left(\frac{\partial U_\theta}{\partial \xi} - \frac{U_\theta}{\xi}\right), \quad \sigma_{z\theta} = \mu\frac{\partial U_\theta}{\partial \eta} \tag{10}$$

Particularly, Eqs. (6) and (7) are rewritten as:

$$\frac{1}{b}\frac{\partial \Sigma_{z\theta}}{\partial \eta} + \sigma^{+}_{r\theta} - \sigma^{-}_{r\theta} = 0 \tag{11}$$

$$\Sigma_{z\theta} = \mu_s \frac{\partial U_\theta}{\partial \eta} \tag{12}$$

Now, we turn our attention to the solution structure of the boundary value problem. Referring to Fig. 1(a), we have a whole field solution expressed in terms of an asymptotic expansion as (e.g., Fan and Widera [6] and Fan et al. [9])

$$F(\xi,\eta) = F_0(\xi,\eta) + \sum_{k=1}^{\infty} c_k e^{-\lambda_k \eta} F_k(\xi) \tag{13}$$

where $F_0(\xi,\eta)$ is the so-called Saint-Venant solution, which is a non-decay solution with respect to z. The rest terms with exponential decay feature are of our interest. In other words, all the λ_k (in an ascending order) in the summation are positive. Particularly, the