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Power loss and load distribution models including frictional effects for spur and helical gears

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ABSTRACT

Friction between meshing gear teeth are amongst the most influential power loss sources in a gearbox near nominal operating conditions. Speed, load and coefficient of friction are some of the most important factors regarding frictional losses in gears. The aim of this work is to introduce gear load sharing models for spur and helical gears taking into account elastic and frictional effects allowing to do more refined estimations of gear friction losses.

1. Introduction

Gear efficiency has been an object of study for many years now [1, 2]. Nowadays with the environmental and sustainability concerns [3] as well as the need to minimize the operating cost of any machine through its life gear efficiency are still a matter of concern.

Höhn et al. [4] presented a study that showed that it is possible to improve gearbox power loss acting on both gear geometry design and rolling bearing selection. Recently Petry-Johnson et al. [5] presented a test methodology to measure spur gear efficiency under high speed and variable torque conditions. The influence of some of the gear design variables were evaluated in the power loss. These results were later used by Chang et al. [6] to validate the power loss model by them developed.

More recently Fernandes et al. [7–9] performed extensive experimental testing and modelling of the power losses generated by three different gear geometries. The rolling bearings model as well as the gear loss models were properly calibrated from simple experimental tests. The same lubricants were later used in another experimental campaign [10] and power loss predictions using the calibrated model were in good agreement with the measurements.

The most common approach to gear efficiency usually considers a constant and average coefficient of friction, an input power and a gear loss factor that depends on gear geometry and load [2]. Having a correct load distribution and friction formulation is then very important to obtain accurate power loss predictions.

Several authors [2, 4, 11–13] already presented formulas based on empirical studies that aim to provide an average and constant coefficient of friction along the path of contact. Recently Xu [14] proposed an experimentally validated formula derived from a very large amount of numerical simulations that can be used to calculate the coefficient of friction along the path of contact in a meshing teeth pair.

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The gear loss factor problem was already studied by several authors in previous works [8]. The formulations that are available in the literature were developed after different assumptions. These formulations usually work quite well for spur gears, but in the case of helical gears some cautions must be taken [8]. Having precise gear loss factor estimations is fundamental to have better gear power loss estimations.

Several works [15–19] just to name a few, considering different approaches already dealt with the load sharing problem in meshing gears. Considering a load distribution taking into account mesh stiffness will lead to a better understanding of not only the power loss along the path of contact, but also allowing more refined calculations of the gear loss factor [20]. It can also contribute to explain the occurrence of certain tooth flank distress phenomena.

In this work the load distribution problem is studied considering rigid and elastic teeth while disregarding dynamic and Hertzian effects. The load dependent Hertzian non-linear effects play a more important role in the mesh stiffness than in the load distribution where the non-linear effect is diminished.

The static rigid model is based on the lengths of the lines of contact and takes advantage of the properties of an approximation of the Heaviside step function to obtain a continuous description of the load distribution.

The static elastic model is based on the minimization of the total potential energy stored in the system. The load balance including friction forces is introduced using a Lagrange multiplier. This solution is a refined version of a previous work already presented by the authors [21].

The static elastic load distribution model was also used to evaluate the gear loss factor of three gear tooth geometries: a spur (C14), an helical gear (H501) and a "low loss" gear (H951) [8].

The main goal of this paper is then to present two different gear load sharing models that are of simple formulation, implementation and also have little to no limitations of application in what concerns spur or helical gear geometries.

2. Gear geometries

In the meshing process the gear teeth are in contact along parallel lines that form a plane which is at every instant normal to the surface of all meshing teeth. This plane, the plane of action, is also tangent to both base cylinders [22].

Table 1 shows the geometric parameters for three different gear geometries, one spur and two helical gear geometries.

Consider that spur gears are the particular case of helical gears when the helix angle is zero. There are three different scenarios regarding the equations that describe the lines of contact along the plane of action:

1. $\epsilon_\beta = 0$
2. $\epsilon_\beta \leq \epsilon_\alpha$
3. $\epsilon_\beta > \epsilon_\alpha$

Fig. 2 show the three different scenarios, i.e., Fig. 2a shows the lines of contact for a spur gear, Fig. 2b shows the lines of contact for an helical gear with $\epsilon_\beta \leq \epsilon_\alpha$ (lines of contact cover the entire tooth flank) and Fig. 2c also shows the lines of contact for an helical gear, but $\epsilon_\beta > \epsilon_\alpha$ (lines of contact never cover all the tooth flank length).

During the meshing process, there may be more than one teeth pair simultaneously engaged. The lines of contact corresponding to more than one teeth pair simultaneously engaged are separated by a distance equal to the transverse base pitch (p_{bt}) from each other.

In the case of spur gears ($\epsilon_\beta = 0$) a meshing teeth pair enters the "active" section of the plane of action and the line of contact has a length that is equal to the face width of the gears (b).

In helical gears the meshing teeth pair gradually enters the path of contact and the length of the contact lines show a linear increase from 0 up to a maximum and then decrease back to 0 as it leaves the active section of the plane of action. This maximum can be either a *plateau* or a peak depending on the relationship between ϵ_β and ϵ_α.

From these considerations, it is possible to define the length and the sum of the lengths of the lines of contact along the active section of the plane of action.

In a spur gear, the contact lines are parallel to the lines that are tangent to the plane of action and the base cylinder as represented in Fig. 2a. In helical gears, the contact lines are at an angle (base helix angle, β_b) with these tangency lines as represented in Fig. 2b and c.

Table 1
Geometrical parameters of the C14, H501 and H951 gears.

Gears		Parameters									
		Z [/]	m [mm]	α [mm]	α [°]	β [°]	b [MM]	x [/]	Ra [μm]	ϵ_i	ϵ_β
C14	Driving	24	4.5	91.5	20	0	14	+0.1715	0.4	0.7224	0
	Driven	16						+0.1817		0.7153	
H501	Driving	30	3.5	91.5	20	15	23	+0.0891	0.4	0.7575	0.5414
	Driven	20						+0.1809		0.7025	
H951 (Low loss)	Driving	57	1.75	91.5	20	15	23	+2.0003	0.4	0.5630	1.0828
	Driven	38						+1.6915		0.3716	