Mechanism and Machine Theory 96 (2016) 127–145

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Computerized design, generation and simulation of meshing and contact of hyperboloidal-type normal circular-arc gears

Houjun Chen *, Xiaoping Zhang, Xiong Cai, Zhilan Ju, Chang Qu, Donghe Shi

School of Mechanical Engineering, Nantong University, Nantong 226019, PR China

ARTICLE INFO

Article history:
Received 14 April 2015
Received in revised form 8 July 2015
Accepted 19 August 2015
Available online 11 November 2015

Keywords:
Wildhaber–Novikov gear
Crossed-axis transmission
Tooth contact analysis

ABSTRACT

The authors proposed an approach for computerized simulation of meshing and contact of misaligned hyperboloidal-type normal circular-arc gears (HNCGs). According to local conjugate theory, the point-contact design method for HNCGs was presented, and the equations of pinion-gear tooth surfaces were deduced. Using the moving frame method, the errors and the variations were quantified, and the error-variation equations were established. Based on Hertz contact hypothesis, the contact region of pinion-gear teeth was determined, and the computerized simulation program for tooth contact was developed. By the visualization of 3D- and 2D-contact regions, the influence of misalignment errors on the shift of the bearing contact has been explored. Numerical examples that illustrate the developed theory were provided.

1. Introduction

Wildhaber recommended helical gears with circular-arc tooth profile in an US patent [1], and Novikov realized this gear by the hobs in the gear manufacturing industry [2]. Though there is a significant difference between the above inventors' ideas, it was finally unified and named as Wildhaber–Novikov gear (for short W–N gear) at an international gearing conference held at Essen of Germany. W–N gear became very popular in the 1960s, and many researchers in USSR, Japan, Germany and China made valuable contributions to this research area. However, due to the development of hardened involute gear, W–N gear has not been found a broad popularization in the industry of Western World. In fact, it has many great advantages, such as high contact strength, low sensitivity to errors and compact structure. In order to reduce the noise and vibration caused by the misalignment, Litvin et al. proposed a novel method to generate W–N gear by “two rigidly connected” tool surfaces with parabolic profiles in internal tangency [3]. Neither single circular-arc tooth profile nor double circular-arc tooth profile, W-N gear was always limited at the parallel-axes gear transmission before 21st century. With the reformation of research technique, Kuo [4] proposed a novel bevel gear with circular-arc tooth profile, and obtained the ideal conditions of interference-free tooth surface by the spherical geometry. Almost simultaneously and independently, Duan [5,6] and Maiki [7] proposed a generating method of W–N spiral bevel gear, and the latter also conducted Tooth Contact Analysis (TCA) to check the tooth contact mark in the running-in test. Tsai and Hsu [8] proposed a kind of spiral bevel gear with circular-arc contact path and tooth profile, and presented a machining method on general milling machine. Gu et al. [9] proposed a new application of double circular-arc tooth profile in the nutation drive, and developed a mathematic model of tooth surface. Directed by the principle of moulding-surface conjugation, Chen et al. developed a theoretical prototype of hyperboloidal-type normal circular-arc gear (HNCG), in which the circular arc tooth profile is extended into the crossed-axes gear transmission [10,11].

---

* Corresponding author.
E-mail address: mrchj365@ntu.edu.cn (H. Chen).

http://dx.doi.org/10.1016/j.mechmachtheory.2015.08.022

Nomenclature

S_i	Coordinate systems rigidly connected to pinion ($i = 1$) and gear ($i = 2$)
O_i	Center of throat circle of datum surface for pinion ($i = 1$) or gear ($i = 2$)
E	Center distance between the axes of pinion and gear
R_i	Throat radius of datum surface for pinion ($i = 1$) and gear ($i = 2$)
N_i	Number of teeth of pinion ($i = 1$) and gear ($i = 2$)
m_n	Normal module
m_{12}	Gear ratio
δ	Inclined angle of the meshing line of directrixes with respect to the pinion axis of rotation
$\boldsymbol{\rho}$	Radius vector of meshing line of directrixes of pinion-gear tooth surfaces
u	Length parameter of meshing line
λ_i	Angle of rotation of pinion ($i = 1$) and gear ($i = 2$) in the process for generation
Σ_{k}	Datum surface of pinion ($k = p$) and gear ($k = g$)
$\boldsymbol{P}_k$	Radius vector of datum surfaces for pinion ($k = p$) and gear ($k = g$)
$\mathbf{R}_k^{(t)}$	Radius vector of theoretical directrix of tooth surface for pinion ($k = p$) and gear ($k = g$)
$\mathbf{R}_k^{(r)}$	Radius vector of real directrix of tooth surface for pinion ($k = p$) and gear ($k = g$)
$\Delta\lambda_i$	Modified angle of directrix of pinion ($i = 1$) and gear ($i = 2$) in the point-contact design
l_i	Transverse offset of center of tooth profile of pinion ($i = p$) and gear ($i = g$)
$\mathbf{e}_1^{(i)}, \mathbf{e}_2^{(i)}, \mathbf{e}_3^{(i)}$	Unit basis vectors of moving frame of directrixes for the tooth surface of pinion ($i = p$) and gear ($i = g$)
$\mathbf{R}^{(i)}$	Radius vector of tooth surface for pinion ($i = 1$) and gear ($i = 2$)
r_i	Radius of active tooth profile for pinion ($i = p$) and gear ($i = g$)
α_n	Normal pressure angle
θ	Profile angle of circular arc tooth profile
φ_i	Angle of rotation of pinion ($i = 1$) and gear ($i = 2$) in the process of meshing
$B(-\lambda_i)$	Rotation group about the axes of pinion ($i = 1$) or gear ($i = 2$), and its calculation rules are listed in the reference [10]
$\mathbf{e}(\lambda_i), \mathbf{e}_1(\lambda_i)$	Circle vector functions about the axes of pinion ($i = 1$) and gear ($i = 2$), and their calculation rules are listed in the reference [11]
Σ_{i}	Tooth surface of pinion ($i = 1$) and gear ($i = 2$)
$\mathbf{r}^{(i)}, \boldsymbol{\alpha}_{s3}^{(i)}$	Radius vector and normal vector of ideal contact point on pinion ($i = 1$) and gear ($i = 2$) tooth surfaces in the conjugation process
$\mathbf{r}^{(i*)}, \boldsymbol{\alpha}_{s3}^{(i*)}$	Radius vector and normal vector of actual contact point on pinion ($i = 1$) and gear ($i = 2$) tooth surfaces in the conjugation process
$k_{s1}^{(12)}, k_{s2}^{(12)}$	Induced principal curvatures between pinion-gear tooth surfaces
$\mathrm{C}_k^{(t)}$	Directrix of theoretical tooth surface for pinion ($k = p$) and gear ($k = g$)
$\mathrm{C}_k^{(r)}$	Directrix of real tooth surface for pinion ($k = p$) and gear ($k = g$) in the point-contact design
$k_{s1}^{(i)}, k_{s2}^{(i)}$	Principal curvatures of tooth surface for pinion ($i = 1$) and gear ($i = 2$)

In the early 1960s, the tooth contact analysis technique has been proposed by Baxter [12], and then a novel and powerful tool was developed to determine the contact characteristic and running quality of spiral bevel and hypoid gear transmissions. Though it is a theoretical analysis, it can substantially reduce the lengthy trial-and-error procedure for the design and development of many gears by the application of the TCA program. In order to obtain a more realistic picture of tooth contact characteristic, the tooth contact analysis under loaded conditions, the loaded tooth contact analysis (LTCA), was developed by Krenzer in the late 1970s, in which tooth deformation and shaft deflection due to loading were taken into account to determine actual geometry

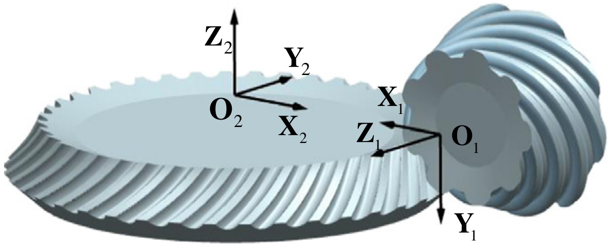

Fig. 1. Hyperboloidal-type normal circular-arc gear drive.