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Analytical approaches to oscillators with nonlinear springs in parallel and series connections

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ABSTRACT

We develop analytical approaches and analyze their range of applicability to oscillators with two nonlinear springs in parallel and series connections. Specifically, we focus our study on systems with hardening springs and cubic nonlinearities. In both cases, three dimensionless parameters govern the oscillator namely $\lambda = k_2/k_1$ and $\epsilon_{1,2} = \varepsilon_{1,2}A^2$. Here, $k_{1,2}$ and $\varepsilon_{1,2}$ are the linear stiffness and the nonlinearity coefficient of both springs, respectively, and A is the amplitude of the position of the mass, in the parallel case, or the deflection of the spring connected to the mass (k_2, ε_2), in the series case. It is found that, in parallel configuration, for $\lambda > 0$ and $0 < \epsilon_{1,2} \leq 1$ the analytical solution gives an excellent approach to the exact solution found numerically. However, in series connection the numerical simulations show that the solution of the oscillator becomes much more complex than in parallel connection, and the analytical approaches work excellently in the ranges $0 < \lambda \leq 1$, $0 < \epsilon_{1,2} \leq 0.1$, and, $0 < \lambda \leq 0.1$, $0 < \epsilon_{1,2} \leq 1$.

1. Introduction

The problem of nonlinear oscillators occurs in many real systems from macro to nano length scales. Therefore, nonlinear oscillators appear in many fields of science as mathematics, physics, mechanics, electronics, chemistry, biology, astronomy, etc. Usually, the mathematical model of a nonlinear oscillator is a strong nonlinear second order differential equation. Such nonlinear equation is very difficult to be solved especially with analytical methods [1–5].

Many effective analytical methods have been suggested to solve basic oscillators with one nonlinear spring, such as the variational method [6–8], homotopy perturbation method [9–13], parameter expansion method [14,15], energy balance method [16–19], harmonic balance method [20–22] or the Hamiltonian method developed by He [23–26], among others. Regarding combined oscillators with linear and nonlinear stiffness in series, Telli and Kopmaz [27] showed that the motion of a mass grounded via linear and nonlinear springs in series leads to a set of differential algebraic equations. They demonstrated that introducing a suitable variable representing the deflection of nonlinear spring, one can obtain a nonlinear ordinary differential equation which can be solved using the Lindstedt method [28,29] and the harmonic balance method. Lai and Lim [30] extended the perturbation and harmonic methods for a nonlinear system combining linear and nonlinear springs in series. The governing equation was linearized and associated with the harmonic balance method to establish new and accurate higher-order analytical approximate solutions. Using homotopy analysis method and homotopy Padé technique to accelerate the convergence rate of the series solution, Hoseini et al. [31] analyzed nonlinear free vibration of conservative oscillators with inertia and static type cubic nonlinearities. Barforoushi et al. [32] applied also the homotopy perturbation method for solving nonlinear free vibration of systems with serial linear and nonlinear stiffness. They

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found that this method is suitable for this kind of oscillator although they showed the limitations of such method for higher degree of freedom systems. Recently, the approximate value of the equivalent rigidity for linear and nonlinear springs in series was obtained by Bayat et al. [33], applying the averaging procedure [34]. Moreover, they showed that the approximate value of the equivalent frequency of vibration of the oscillator corresponds to the value which is obtained with the Hamiltonian approach.

In order to validate the analytical approaches commented above, the numerical solutions of the nonlinear oscillators are also necessary. One of the most efficient methods to integrate numerically strong nonlinear second order differential equations is the fourth-order Runge–Kutta method [19,35–37]. However, there are alternative methods even more accurate for solving numerically such equations. Razzaghi and Elnagar [38] used the pseudo spectral method to find the numerical solution of a Duffing oscillator. Arikoglu and Ozkol [39] resolved numerically different nonlinear integro-differential equations using the differential transform method (DTM). They shown that the DTM method is a very fast convergence, precise and cost efficient tool for solving these nonlinear equations. Using the Laplace transform and the Padé approximations, Momani and Ertürk [40] proposed a new method to capture with accuracy the frequency of the response of nonlinear oscillators.

Although the numerical methods can be very accurate tools to find nonlinear solutions, they have the drawback to be valid only for certain numerical parameters. Thus, the numerical results are applicable for solving different technical problems, quantitatively, although they are not enough for a deep qualitative analysis of the problem. Because of this limitation, we need the analytical approach to the solution of the nonlinear problem, which must be suitable for discussion.

Therefore, the main objective of the present work is to extend the application of the averaging procedure, based on the equivalent rigidity, to find analytical approaches to oscillators with two nonlinear springs in both parallel and series connections. Furthermore, it is also proposed a suitable dimensionless analysis to identify easily the application ranges of the analytical approaches found. This paper begins with the deduction of the analytical approaches to oscillators with nonlinear springs in parallel and series, which are developed in Sections 2 and 3, respectively. The comparisons of the analytical approaches with numerical solutions, as well as their ranges of applicability, are given in Section 3. The main conclusions and suggestions for future work are outlined in Section 4, while the numerical aspects are detailed in Appendix A.

2. Analytical approach to an oscillator with nonlinear springs in a parallel configuration

A nonlinear spring has a nonlinear relationship between force and displacement. In the current work, we consider hardening springs with a nonlinear stiffness

$$k(y) = k_0\left(1 + \varepsilon y^2\right), \tag{1}$$

where y is the net deflection of the nonlinear spring, k_0 is the constant stiffness, and ε is the nonlinearity coefficient. This kind of nonlinear springs appears in the so-called Duffing equation which has been widely studied [23,41]. The case $\varepsilon > 0$ corresponds to a hardening spring while $\varepsilon < 0$ indicates a softening one, see Fig. 1.

Therefore, the restoring force of the nonlinear spring is

$$F(y) = -k(y)y = -k_0\left(y + \varepsilon y^3\right), \tag{2}$$

which has a linear contribution $-k_0 y$ and a nonlinear one $-k_0 \varepsilon y^3$.

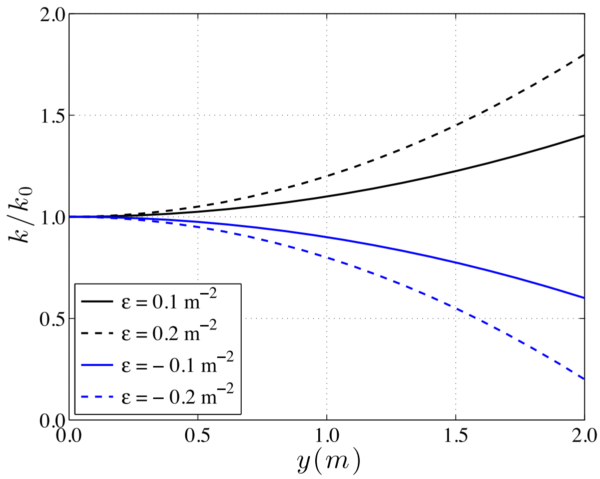

Fig. 1. Dimensionless nonlinear stiffness for different values of ε.