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Mobility of symmetric deployable structures subjected to external loads

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Deployable structures can exhibit remarkable and continuous geometric transformations, however, they are likely to be rigid under certain external loads. This study adopts group theory to evaluate the mobility of symmetric deployable structures under external loads. Mobility analysis is expressed as determining the orthogonality of internal mechanism modes and external loads. Based on the symmetry groups, both the mechanism modes and external loads are associated with specific symmetry subspaces. Thus, it can be evaluated whether the external loads stiffen all the internal mechanism modes. Illustrative examples on pin-jointed structures and over-constrained mechanisms are given to verify the proposed method. It turns out that the product of the internal mechanisms and external loads is equivalent to that of the mechanisms and loads in the symmetry subspaces associated with different irreducible representations. A deployable structure will be immobile under external loads if symmetry order of the loads is higher than that of the mechanisms. In addition, the structure will be immobile, if the internal mechanisms and the external loads are equisymmetric and orthogonal to each other. The conclusions agree with published results, and need much fewer computations. The proposed method is efficient and applicable to most symmetric deployable structures.

1. Introduction

More and more novel kinematically indeterminate structures appear in recent years, such as tensegrity structures, prestressed cable-strut systems, and deployable structures. These structures have internal mechanism modes and thus their mobility is uncertain. The mobility mentioned in this study is the kinematic characteristic of a structure, which determines whether the structure exhibits remarkable and continuous geometric transformations without inducing strain. According to the definition of mobility, deployable structures are generally mobile and can be taken as finite mechanisms. The geometric transformation of a deployable structure (deployment or folding) is desirable, which allows the structure to be easily folded in size for storage or transportation. More importantly, such a structure can be stable through prestressed or locked by additional actions, when it is required for maintaining stable equilibrium or reaches designed configuration (umbrella is a simple example). In addition, although deployable structure exhibits transformable geometry in the process of deployment or folding, it could bear certain external loads with a specific configuration. Hence, mobility of deployable structures under external loads becomes one important aspect of stability and kinematic analysis.

Earlier studies were mainly focused on mobility and stability analysis of prestressable structures such as tensegrity structures and cable-strut structures [1–6]. As kinematically indeterminate structures, these structures contain internal mechanism modes in the

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initial configuration, but become stable after prestressed. Tarnai and Szabó [7] discussed the definition and stability of kinematically indeterminate pin-jointed structure. Subsequently, Pellegrino and Calladine [8] presented the equilibrium matrix theory for pin-jointed structures, and provided a criterion to determine the geometric stability of a structure [9]. The criterion is systematic, and filled a gap in the well-known Maxwell's rule. It could evaluate not only the geometric stability, and also the static and kinematic indeterminacy for a pin-jointed structure. However, the criterion is a necessary but not sufficient stability condition [3,4,10]. Based on the constraint equations and the statical kinematic stiffness matrix, Kuznetsov [11] discussed the kinematic indeterminacy and mobility of first-order infinitesimal mechanisms. Vassart et al. [12] investigated the infinitesimal mechanisms using the energy method. Built on the geometric nonlinearity theory and the energy criterion, Deng et al. [3] proposed the classification for stability conditions of pin-jointed bar assemblies. Thus, the mobility problem of a pin-jointed structure was transformed into consideration of high-order variations of the potential energy. Besides, some studies have pointed out that the stability of a structure depends on its geometric topology, member stiffness, kinematic constraints, and external loads [4,13,14].

In essence, a deployable structure belongs to the mechanical system. Unlike infinitesimal mechanisms [9] or prestressed structures [6,15], deployable structures cannot maintain stable equilibrium through initial prestresses. The structures allow finite and continuous motions [16], and many of them are symmetric [17,18]. Recent studies have successfully utilized group theory to deal with static stability, bifurcation, and vibration of symmetric space structures [19–22]. Based on the irreducible representations of symmetry groups, Guest and Fowler [23] proposed a symmetry-extended mobility rule for symmetric mechanisms. According to the mobility rule, symmetries of mobility and internal mechanism modes could be predicted. Meanwhile, most symmetric finite mechanisms could be effectively identified, for example, cyclically symmetric pin-jointed structures [24] and highly symmetric two-orbit switch-pitch structures [25]. In addition, mobility and kinematic of finite mechanisms could be investigated using screw theory [26,27]. Wei and Dai [28] discussed the mobility of Hoberman switch-pitch ball, which retains tetrahedral symmetry during folding. Ding et al. [29] studied the mobility and singularity of a deployable mechanism with prismatic symmetry by screw theory. The symmetric mechanism could present radial motions, and has potential application in the aerospace field. Chen and You [30] designed a type of two-fold symmetric 6R foldable frame. They discovered the bifurcations of the foldable structure utilizing the singular value decomposition of the Jacobian matrix. Later, Viquerat et al. [31] introduced polynomial continuation to design a plane symmetric 6R foldable ring. Lengyel and You [32] analyzed the compatibility conditions of mechanisms with one degree-of-freedom using the elementary catastrophe theory. The bifurcations along motion paths were illustrated through some finite mechanism examples.

Nevertheless, few studies investigate the mobility of finite mechanisms under external loads. In fact, the external loads play an important role in the mobility and stability of deployable structures. It is essential to note that deployable structures could be rigid under certain external loads [3,5,33]. The tangent stiffness of a deployable structure is affected by the internal forces in members, which are induced by external loads. In this case, the structure has a potential ability to maintain the stability of its equilibrium configuration. Lu et al. [33] proposed a criterion to evaluate the mobility and equilibrium stability of pin-jointed mechanisms under external loads. If immobile, the mechanisms could retain certain shapes to keep static equilibrium.

Here, we propose an efficient symmetry method for evaluating the mobility of symmetric deployable structures under external loads, where the kernel is to determine the orthogonality of the internal mechanisms and external loads. Based on group theory and the great orthogonality theorem, both the mechanisms and loads will be associated with specific symmetries. Accordingly, it will be much easier to identify whether the external loads stiffen all the internal mechanisms. Illustrative examples on pin-jointed deployable structures and over-constrained mechanisms will be presented, to verify the feasibility and efficiency of the proposed method.

2. Mobility of a structure under external loads

For a structure with n generalized nodal coordinates and subjected to c kinematic constraints $F_k = 0$ ($k = 1, \cdots, c$), the potential energy is a function of the generalized nodal coordinates $\boldsymbol{X}$ and the Lagrange multipliers $\boldsymbol{\Lambda}$ [33,34]:

$$\boldsymbol{\Pi}_R(\boldsymbol{X},\ \boldsymbol{\Lambda}) = -\sum_{i=1}^{n} P_i(X_i - X_i^0) + \sum_{k=1}^{c} \Lambda_k F_k \tag{1}$$

where P_i ($i = 1, \cdots, n$) are the external nodal loads, X_i^0 and X_i are the i-th generalized coordinates at the initial and deformed configurations. Λ_k is the k-th Lagrange multiplier, and could be regarded as the internal force of members. $F_k(X_1, \cdots, X_n) = 0$ is the k-th kinematic constraint function. From the energy point of view, the equilibrium state could be established by the condition of potential energy minimization, given as:

$$\delta\boldsymbol{\Pi}_R(\boldsymbol{X}, \boldsymbol{\Lambda}) = \boldsymbol{0} \tag{2}$$

Under certain external loads, a structure will reach an equilibrium state by adjusting its form and the internal forces. Mobility analysis on a deployable structure under external loads is actually a series of form-finding problems on the equilibrium configuration determined by some variables (for example, the nodal displacements). When a deployable structure is in an equilibrium state under nonzero external loads $\boldsymbol{P}$, it satisfies:

$$\delta\boldsymbol{\Pi}_R(\boldsymbol{X}, \boldsymbol{\Lambda}) = \left[\left(\frac{\partial \boldsymbol{\Pi}_R}{\partial X_i}\right)^{\mathrm{T}} \quad \left(\frac{\partial \boldsymbol{\Pi}_R}{\partial \Lambda_k}\right)^{\mathrm{T}} \right] \begin{bmatrix} \delta X_i \\ \delta \Lambda_k \end{bmatrix} = \boldsymbol{0}. \tag{3}$$