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Geometrical analysis of a quadrilateral rotary piston engine

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ABSTRACT

This paper presents geometrical analysis of a novel rotary device suitable for operation as an engine, an expander and a pump. The device comprises a stator having an elongated oval-shaped chamber encompassing a quadrilateral deformable rotor. The analysis includes derivation of the closed-form equations describing the inner stator profiles, compatible rotor-segment profiles, volume variation, compression ratio, and displacement volume. The study shows the pivotal importance of the chamber eccentricity ratio and inner offset parameters on the performance of the device. Verification of the analytical results was made by comparing them with those obtained by numerical integration at selected points.

1. Introduction

The evolution of mechanisms and machines covering the aspects of theory and design has been addressed in [1–3]. This paper presents a theoretical study for an innovative rotary device referred to as rhomboidal rotary machine (RRM) or quadrilateral rotary machine (QRM). The device is a positive displacement one and configurable to operate as a power-producing device such as an expander or an engine equivalent to four-stroke internal combustion engine, as depicted schematically in Fig. 1, or as a power-consuming device such as a pump or a compressor.

One class of rotary devices, which bears some relevance to the present device, is termed planetary rotation machines or trochoidal-type machines [4]. These machines comprise two components, a rotor and a stator (i.e. a chamber), whereby the rotor in the stator executes a general plane motion (i.e. a combination of rotation and curvilinear translation along a circular orbit) whereas the rotor is continuously in contact with the inner stator at a number of points, and hence defines a multiplicity of varying volume sub-chambers. In such machines, one of the two components (i.e. rotor or stator) comprises a profile mathematically defined by a trochoid, and the other respective component (i.e. rotor or stator) is referred to as an envelope. The envelope comprises a family of curves that include limiting cases referred to as conjugate envelopes. Examples of such devices include trochoidal-type rotary pump and Wankel machine [5].

The present device under consideration, QRM, primarily differs from the trochoidal-type machines, firstly, in the adoption of a deformable rotor as opposed to rigid rotor and, secondly, in the execution of a pure rotational motion of the rotor assembly as opposed to a combination of rotation and curvilinear translation motion (Table 1).

According to the above table, the pure rotational motion and gearless transmission represent the important advantages of QRM over a Wankel machine. Historically, the earliest reporting of the concept of QRD was made in a US patent by E.H. Werner, 1902 [6], and this followed by a number of patents [7–11]. Literature search on the subject of QRM yields no theoretical work on the subject, and thereby, the present study, according to author's knowledge, is the first theoretical investigation on the subject.

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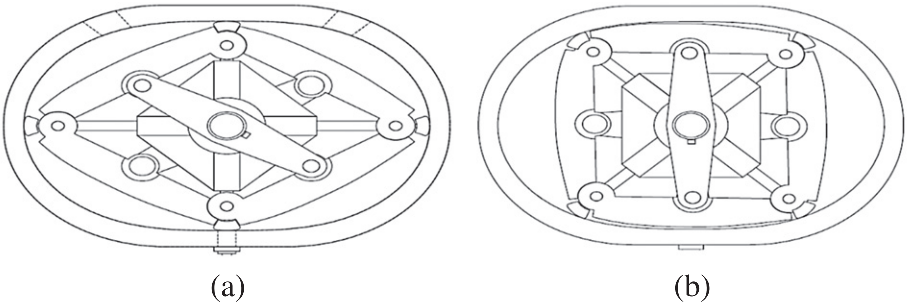

Fig. 1. Quadrilateral rotary device (a) rhombus orientation (b) square orientation.

This study focuses on the geometrical aspects of QRM, as presented on the following sections. Section 2 derives the mathematical equation for the inner stator profile. Section 3 identifies potential rotor segment profiles compatible with inner stator profiles. Section 4 derives closed-form equations of volume variation for different classes of rotor-stator profiles. Sections 5 and 6 derive closed-form expressions for compression ratio and displacement volumes. Section 7 addresses the effect of apex seal protrusion on performance. Section 8 discusses the results and provides numerical verification of the results. Lastly, Section 9 summarizes important findings and suggests topics for future work.

2. Stator geometry

Although the general oval-shaped profile is well known in the literature for such devices, no exact mathematical definitions were reported upon reviewing the literature. Thus, the objective of this section is to derive the exact mathematical formula for such profiles. The selected profile must permit the inclusion of deformable quadrilateral rotor that rotates without seizure while making constant contact at four points. For this purpose, consider a symmetrical oval-shaped profile, shown schematically in Fig. 2, oriented with major axis along the horizontal x-axis and having a characteristic length, R, and eccentricity, e, such that major and minor radii are equal to $R + e$ and $R - e$, respectively. Using dimensionless polar coordinates, (r, θ), where $r = r'/R$ and θ measured from the horizontal positive in counterclockwise direction, and the eccentricity parameter, k, defined as $k = e/R$. Now, the quadrangular rotor with four vertices as represented schematically by a rhombus shape with four equal sides and two perpendicular diagonals is equivalent to a set of four mirror-imaged right angle triangles. For the purpose of derivation, it is sufficient to consider a single right angle triangle as depicted in the figure. Thus, considering a rotating right-angled triangle having a fixed dimensionless hypotenuse of length, L (i. e. $L = L'/R$), rotating in a counterclockwise direction about its right angle vertex, 0, coinciding with the center of the oval-shaped profile with both leading vertex p_l and trailing vertex p_t tracing the oval-shaped profile. Letting the angular positional vector of the triangle coincide with a line bisecting the 90° vertex such that when $\theta = 0$, the right angle hypotenuse aligns vertically, and when $\theta = \pi/2$ the hypotenuse, it aligns horizontally. Hence, at any angular position, the triangle is mathematically described by the Pythagorean relation as

$$r_l\left(\theta+\frac{\pi}{4},k\right)^2+r_t\left(\theta-\frac{\pi}{4},k\right)^2=L^2, \tag{1}$$

where r_l and r_t represents the leading and trailing sides of the rotating triangle. Among the different available oval-shaped profiles, the one satisfying the above equation is to be derived in the following paragraph.

Considering a general oval-shaped profile described in polar coordinates by the general equation

$$r'(\theta)=\sqrt{C_1+C_2\cos(\theta)^2},$$

Table 1
Comparison between QRM and Wankel machine.

	QRM	Wankel
Chamber shape	Epitrochoid	Epitrochoid
Rotor shape	Quadrangle	Triangular
Rotor flexibility	Deformable	Fixed
Rotor speed	Equal shaft speed	One-third shaft speed
Rotor motion	Pure rotation	A combination of rotation plus curvilinear translation
Power transmission	Gearless	Internal gear