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Establishing the limiting conditions of operation of magneto-rheological fluid dampers in vehicle suspension systems

Raghavendra D. Naik, Pravin M. Singru *

Mechanical Engineering Group, Birla Institute of Technology and Science, Pilani, Goa Campus, Zuarinagar, Goa 403726, India

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ABSTRACT

This paper deals with dynamical behavior of a nonlinear suspension system. We examine chaotic motion in a vehicle suspension system with hysteretic nonlinearity excited by a road profile. A one degree of freedom quarter-car model with nonsymmetric potential is investigated. The Melnikov criterion is used to study the intersection of stable and unstable manifolds and transition to chaos for the system. The condition for chaotic vibration is found using a Melnikov function. Chaotic motion also is indicated by a bifurcation diagram and Lyapunov exponents.

1. Introduction

Magneto-rheological and electro-rheological fluids usually consist of pure and soft iron particles in an insulative carrier liquid. The essential feature of magneto-rheological and electro-rheological fluids is their ability to reversibly change states from a viscous fluid to a semisolid or even solid with controlled yielding strength within a few milliseconds when they are subjected to controlled magnetic/electric fields. Compared with electro-rheological fluids, the viscosity control of magneto-rheological fluids requires a power supply of low voltage only. Thus magneto-rheological fluid appears to be an effective alternative to electro-rheological fluids in use as controllable dampers for vehicle suspension systems. The dislocation movement and plastic slipping among molecular chains or crystal lattices consume energy such that the restoring force of a magneto-rheological fluid damper always delays the input displacement or velocity. This phenomenon of energy dissipation is known as hysteresis. Multivalued and non-smooth hysteresis will lead to many complicated behaviors such as bifurcation and chaos Li et al. (2004).

Litak et al. (2008) investigated a possible chaotic motion in a nonlinear vehicle suspension system which is subjected to multi-frequency excitation from a road surface. Litak et al. (2008) investigated global homoclinic bifurcation and transition to chaos in the case of a quarter-car model excited kinematically by a road surface profile.

In this paper an existing quarter-car model is studied with a semi-active suspension system and the addition of a gravitational term. The condition of chaotic vibration is proved by using Melnikov method Li et al. (2004). The chaos in the system is shown by using a bifurcation diagram and Lyapunov exponents Wolf et al. (1985). The existence of chaos in the system also is illustrated via periodic phase plane plot (periodic Poincaré sections) and time history.

2. The quarter-car model

The one degree of freedom quarter-car model with hysteretic nonlinear damping is studied as shown in Fig. 1. The equation of motion of the system is given by

* Corresponding author. Tel.: +91 832 2580338; fax: +91 832 2557033.
E-mail address: rdn@bits-goa.ac.in (P.M. Singru).

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Nomenclature

M	the mass of the body
C_2	nonlinear damping coefficient
g	acceleration due to gravity
x	the body's vertical displacement
Ω'	excitation frequency
k_2	nonlinear stiffness
ω	natural frequency
C_1	linear damping coefficient
k_1	suspension stiffness
x_0	the road excitation
A	amplitude
y	relative displacement

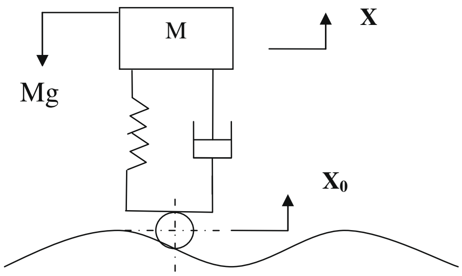

Fig. 1. The quarter-car model with nonlinear damping and stiffness.

$$M\ddot{x} + k_1(x - x_0) + Mg + F_z = 0 \tag{1}$$

where F_z is hysteretic nonlinear damping force of magneto-rheological fluid damper;

Now let $y = x - x_0$, $x_0 = A\,\mathrm{Sin}\,\Omega' t$ and $F_z = k_2(x - x_0)^3 + c_1(\dot{x} - \dot{x}_0) + c_2(\dot{x} - \dot{x}_0)^3$

Eq. (1) can be written as

$$\ddot{y} + \omega^2 y + B_1 y^3 + B_2 \dot{y} + B_3 \dot{y}^3 = -g + F\mathrm{Sin}\,\Omega' t \tag{2}$$

where $\omega^2 = \frac{k_1}{M}$, $B_1 = \frac{k_2}{M}$, $B_2 = \frac{C_1}{M}$, $B_3 = \frac{C_2}{M}$, $F = A\Omega'^2$.

For simulation the following values given by Litak et al. (2007) are considered.

$M = 240$ kg, $k_1 = 160{,}000\ \mathrm{N\,m^{-1}}$, $k_2 = -300{,}000\ \mathrm{N\,m^{-3}}$, $C_1 = -250\ \mathrm{N\,s\,m^{-1}}$, $C_2 = 25\ \mathrm{N\,s^3\,m^{-3}}$.

The corresponding dimensionless equation of motion can be written for a scaled time variable $\tau = \omega t$ as

$$\ddot{y} + y + ky^3 + \alpha\dot{y} + \beta\dot{y}^3 = -g' + A\Omega^2\mathrm{Sin}(\Omega\tau) \tag{3}$$

where

$$k = \frac{B_1}{\omega^2} = \frac{k_2}{k_1}; \quad \alpha = \frac{B_2}{\omega} = \frac{C_1}{\sqrt{k_1 M}}; \quad \beta = B_3\omega = C_2\sqrt{\frac{k_1}{M}}; \quad \Omega = \frac{\Omega'}{\omega} \quad \text{and} \quad g = \frac{g'}{\omega^2} \tag{4}$$

Let $\varepsilon = \alpha$, Substituting in Eq. (3) one obtains

$$\ddot{y} + y + ky^3 + \varepsilon(\dot{y} + c\dot{y}^3 + bg' - f\mathrm{Sin}(\Omega\tau)) = 0 \tag{5}$$

Eq. (5) in state space form can be written as

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= -y_1 - ky_1^3 + \varepsilon(-y_2 - cy_2^3 - bg' + f\mathrm{Sin}(\Omega\tau)) \end{aligned} \tag{6}$$

where

$$c = \frac{c_2\omega^2}{c_1}, \quad b = \frac{M\omega}{c_1}, \quad f = \frac{M\omega A\Omega^2}{c_1} \tag{7}$$