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# A network approach to mechanisms and machines: Some lessons learned

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ABSTRACT

This is essentially a review paper describing progress made in treating mechanisms and machines as networks. Some of the terminology that is helpful to this approach is explained. Relevant elements of graph theory are mentioned. The original aim was to find a robust procedure for finding the instantaneous relative motion of all pairs of bodies within a kinematic chain. The manner in which this was achieved produced several other results that have found unanticipated applications. These are mentioned and publications are cited. Lessons have been learned and these are discussed in Section 11.



## 1. Introduction

The author is glad of this opportunity to thank Erskine Crossley for his many acts of kindness and generosity and to join with others to pay tribute to the work he has done for IFToMM and as editor of the *Journal of Mechanisms*, the forerunner of this journal. In particular, the author can bear witness to the many contributions Erskine Crossley made to good international relations. But this is a technical paper and so it is appropriate to explain the stimulus Erskine Crossley provided that led to the research reviewed in this paper.

Erskine Crossley was the first to mention graph theory in the authors presence. Graph theory [1,2] is a branch of topology concerned with the interconnections within a network of objects. Graph theory has found many applications; most relevant to this paper are applications in electrical network theory, more frequently called electrical circuit theory.

Mechanism and machines can be thought of as coupling networks. Waldron [3] provides rules that apply to couplings arranged in series and in parallel. Like electrical networks, indirect couplings containing cross bracing pose special problems [4]. Baker [5] proposed a simple example that has subsequently proved well-suited as a demonstration for theories that have followed. One solution [6] required the adaptation of Kirchhoff's voltage law. Subsequent work [7–12][1] has led to the adaptation of Kirchhoff's current law as well, and two virtual power equations that use matrices that are identical to those needed for the adaptations of Kirchhoff's laws except for being transposed. All four equations are reproduced in this paper; the adaptations of Kirchhoff's laws Eqs. (1) and (2) in Section 7.2 and the virtual power Eqs. (3) and (4) in Section 8.2. Several applications have been found for the equations [13–31]; further details are provided in Section 10.

## 2. Couplings

Central to the network approach described in this paper is the *coupling*. This term is applied to any means by which an *action* can be transmitted between two bodies that are sufficiently stiff to be regarded as rigid. Furthermore, a coupling must be capable

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[1] Online versions of papers [8,10–12,17,32] can be found from the website of Loughborough University, UK: search for Library, Institutional Repository, Author, Davies, T.H.

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Nomenclature

| | |
|---|---|
| $a$ | the rank of the network unit action matrix $\left[\hat{\mathbf{A}}_N\right]_{dk,C}$ |
| $b_{ij}$ | the element in row $i$, column $j$, of circuit matrix $[\mathbf{B}_M]_{l,F}$ |
| $c$ | degree of constraint of a direct coupling |
| $c_{ij}$ | degree of constraint of bodies $i$ and $j$ of a coupling network |
| $C$ | gross degree of constraint of a coupling network, $\Sigma c$ |
| $C_N$ | nett degree of constraint of a coupling network |
| $d$ | minimum order of the screw system, $1 \leq d \leq 6$ |
| $e$ | number of couplings in a coupling network and edges of coupling graph $G_C$ |
| $f$ | gross degree of freedom of a direct coupling |
| $f_{ij}$ | degree of freedom of bodies $i$ and $j$ of a coupling network |
| $F$ | gross degree of freedom of a coupling network, $\Sigma f$ |
| $F_N$ | nett degree of freedom of a coupling network |
| $k$ | number of independent cutsets of a graph |
| $l$ | number of independent circuits (loops) of a graph |
| $m$ | the rank of the network unit motion matrix $\left[\hat{\mathbf{M}}_N\right]_{dl,F}$ |
| $n$ | number of bodies in a coupling network and nodes of coupling graph $G_C$ |
| $q_{ij}$ | the element in row $i$, column $j$, of cutset matrix $[\mathbf{Q}_A]_{k,C}$ |
| $\{r, s, t; u, v, w\}$ | motion screw components in ray-coordinates |
| $\{R, S, T; U, V, W\}$ | action screw components in axis-coordinates |

*Vectors*

| | |
|---|---|
| $[\mathbf{A}_l]_{dl}$ | $dl$ action components for all $l$ circuits |
| $[\mathbf{M}_k]_{dk}$ | $dk$ motion components for all $k$ cutsets |
| $[\varphi]_C$ | magnitudes of $C$ action screws |
| $[\psi]_F$ | magnitudes of $F$ motion screws |

*Matrices*

| | |
|---|---|
| $\left[\hat{\mathbf{A}}_D\right]_{d,C}$ | unit action matrix of the direct couplings of a coupling network |
| $\left[\hat{\mathbf{A}}_N\right]_{dk,C}$ | network unit action matrix of a coupling network N |
| $[\mathbf{B}_i]_{F,F}$ | diagonal matrices with diagonal elements corresponding to row $i$ of $[\mathbf{B}_M]_{l,F}$; in practice identification is by the circuit label, e. g. $[\mathbf{B}_b]_{F,F}$ for circuit b |
| $[\mathbf{B}_M]_{l,F}$ | circuit matrix of motion graph $G_M$ |
| $\left[\hat{\mathbf{M}}_D\right]_{d,F}$ | unit motion matrix of the direct couplings of a coupling network |
| $\left[\hat{\mathbf{M}}_N\right]_{dl,F}$ | network unit motion matrix of a coupling network N |
| $[\mathbf{Q}_i]_{C,C}$ | diagonal matrices with diagonal elements corresponding to row $i$ of $[\mathbf{Q}_A]_{k,C}$; in practice, identification is by the cutset label, e. g. $[\mathbf{Q}_a]_{C,C}$ for cutset $a$ |
| $[\mathbf{Q}_A]_{k,C}$ | cutset matrix of action graph $G_A$ |

of being disassembled without resorting to cutting. This means that welded and riveted joints are not regarded as couplings, nor are joints formed by adhesion. Action is a term that is sometimes used [11,12,32] as shorthand for a wrench on a screw of any pitch, including a pitch that is zero, namely a force, and a pitch that is infinite, namely a torque. The coupling could be either direct, indirect or a hybrid comprising direct and indirect couplings in parallel. Except where it is necessary to make a distinction, all couplings mentioned are direct couplings. The term coupling is chosen as the name of a superset comprising passive and active couplings, the latter providing sinks or sources of power. Examples of couplings of both kinds have been listed [10]. Important subclasses of passive couplings mentioned in this paper are contact couplings, often referred to as kinematic pairs, and elastic couplings.

As well as the capability of transmitting an action, many couplings also permit relative motion of the bodies they couple. Motion is a term sometimes used [11,12,32] as shorthand for the first time derivative of displacement, geometrically described as a twist rate on the screw of any pitch, including a pitch that is zero, namely an angular velocity, and the pitch that is infinite namely translational velocity. A coupling is characterised by two screw systems [33], a $c$-system of actions that can be transmitted and an $f$-system of motions that can be allowed, and:

$$c + f = d,$$