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On the number synthesis of kinematic chains

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ABSTRACT

Number synthesis of kinematic chains is one of the major steps in the structural synthesis or conceptual design of mechanisms. The work presents a comprehensive literature review on the number synthesis of kinematic chains of mechanisms. The terminology and definitions regarding kinematic chains in literature are introduced first. Then, various methods for the number synthesis of kinematic chains are presented and discussed. Finally, the numbers of kinematic chains with up to 15 links and 7 degrees of freedom are listed.

1. Introduction

Traditionally, the creation of mechanisms to satisfy given requirements and constraints relies on the knowledge, experience, inspiration, imagination, intuition, and/or ingenuity of the designers. Many design concepts are available in references and handbooks for adaption as new designs or the modification of existing mechanisms. Two major systematic approaches, for the creative design, conceptual design, or structural synthesis of mechanisms, have been developed in the past decades. One approach is based on the concept of generalization and specialization [1–4], while the other involves the separation of the structure and the function [5,6]. In both methodologies, the generation of atlas of kinematic chains and/or graphs is a major step of the design process.

In order to describe the relationship between links and joints in a mechanism, the topological structure of a mechanism (*TSM*) should be identified. The *TSM* is characterized by its types and numbers of links and joints, and the incidences between them. In addition, it can be represented by its mechanism topology matrix (*MTM*) [4]. An *MTM* can be transformed into its corresponding kinematic chain or graph. Furthermore, the kinematic chains of mechanisms can be classified into different types, e. g., with simple joints or multiple joints, fractionated or non-fractionated, degenerate, rigid chains, and so on.

In the past long years, many scholars focused on the study of the number synthesis, structural synthesis, structural analysis, isomorphism detection problem of kinematic chains, and some on the literature review of kinematic chains [7–11]. This work carries out an exhaust study on the number synthesis of kinematic chains with simple joints.

2. Terminology and definitions

In what follows terminology and definitions are introduced for clearing the concepts regarding kinematic chains in various publications [4,12].

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2.1. Graph theory

The topological structures or structure synthesis of mechanisms can be represented utilizing graph theory. In this section, some basic concepts and terminology of graph theory are introduced and defined.

2.1.1. Graphs, vertices, and edges

A graph G of order p consists of a finite nonempty set $V = V(G)$ of p vertices together with a specified set E of q unordered pairs of distinct vertices. A pair $X = \{V_1, V_2\}$ of vertices in E is called an edge of G, and X is said to join V_1 and V_2. The vertices V_1 and V_2 are adjacent and vertex V_1 and edge X are incident with each other. A graph with p vertices and q edges is called a (p, q) graph. Graphically, a vertex is symbolized by a small dot and an edge by a line. For the (6, 7) graph shown in Fig. 1(a), vertices 2 and 4 are adjacent, and vertex 6 is incident to edges a and g.

2.1.2. Walks

A walk W of a graph G is an alternating sequence of vertices and edges, $W = (V_1, E_1, V_2, E_2, \ldots, V_i, E_i)$, beginning and ending with vertices, in which each edge is incident with the two vertices immediately preceding and following it. A walk W may also be simply denoted by a sequence of vertices $W = (V_1, V_2, \ldots, V_i)$, whose consecutive vertices are adjacent. For the (6, 7) graph shown in Fig. 1(a), (V_1, V_6, V_2, V_5), $(V_1, V_5, V_2, V_4, V_3)$, and $(V_1, V_6, V_2, V_4, V_3)$ are walks.

2.1.3. Paths and loops

A walk of a graph is defined as a path if the vertices in the graph are distinct, i.e., each vertex of the path is unique. The length of a path is defined as the number of edges in the path. Furthermore, a vertex-path is defined as a path that begins and ends with a vertex, and an edge-path is defined as a path that begins and ends with an edge. For example, the sequence 1, a, 6, g, 2, e, 4 shown in Fig. 1(a) is a path with a length of 3. Besides, a closed walk is defined as a loop if the n vertices are distinct for $n \geq 3$. For the closed walk V_1, V_6, V_2, V_5, V_1 shown in Fig. 1(a) is a loop (L_1).

2.1.4. Isomorphism

Two graphs, G and H, are the same and recognized as isomorphic if there is a one to one correspondence between their vertex sets which preserves adjacency. For every vertex in graph G, there can be found a corresponding vertex in graph H. For the two (6, 7) graphs shown in Fig. 1(a) and (b), since they have the same adjacency, they are isomorphic graphs.

2.1.5. Planar graphs and non-planar graphs

In graph theory, a planar graph is a graph which can be drawn on a plane surface such that edge-crossing does not occur. For instance, the (6, 7) graph shown in Fig. 1(a) is a planar graph, while the (7, 12) graph shown in Fig. 2 is a non-planar graph.

2.1.6. Blocks and planar blocks

In graph theory, a block is a maximal non-separable sub-graph which is connected, nontrivial, and without any cut vertices. In addition, a planar block is a block which can be embedded in a plane with no edge-crossings. A planar block with p vertices and q edges is referred to as a (p, q) planar block. For example, the graph shown in Fig. 1(a) is also a (6, 7) planar block.

2.1.7. Contracted graphs

A contracted graph is a graph comprising only vertices with more than binary vertices and is obtained by contracting all binary vertices until no binary vertices exist in the graph. For example, the (8, 10) graph shown in Fig. 3(a), its corresponding contracted graph is shown in Fig. 3(b).

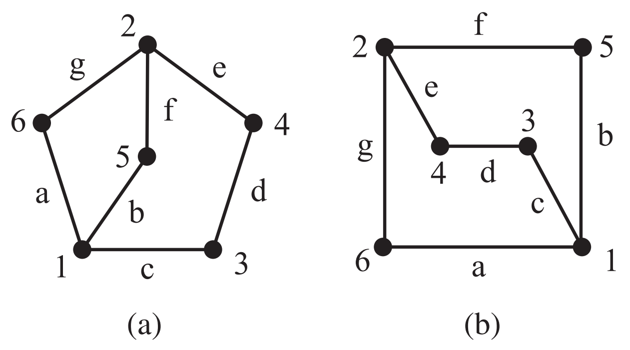

Fig. 1. Two isomorphic (6, 7) graphs.