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Thermal bending analysis of shear-deformable laminated anisotropic plates by the GDQ method

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ABSTRACT

The generalized differential quadrature (GDQ) method was used to determine the interlaminar stresses and deflections in a laminated rectangular anisotropy plate under thermal bending involving the effect of shear deformation. We obtained the non-dimensional stresses and transverse center deflection in cross-ply and angle-ply anti-symmetric, anisotropic laminates subjected to thermal load with sinusoidal temperature distribution. We found that the shear deformation has significant effects on the stresses and deflections for laminated anisotropic plate with moderately side-to-thickness ratio under thermal load and bending state.

1. Introduction

In the 1970s, many researchers (Bellman and Casti, 1971; Bellman, 1973; Bellman and Kashef, 1974; Bellman et al., 1972; Bellman and Roth, 1979a,b) introduced the differential quadrature (DQ) method and provided the computation method for the studies in the fields of engineering and physical sciences. The finite element method (Reddy and Hsu, 1980) and YNS shear deformation theory (Yang et al., 1966) have been used to study the effects of shear deformation and anisotropy on layered composite plates subjected to thermal and mechanical loads. The GDQ method (Shu and Richards, 1992) was firstly presented in 1992. The numerical method of GDQ (Bert et al., 1989; Shu and Du, 1997; Hua and Lam, 1998; Liew and Teo, 1998) have been used and presented in many researches for the behaviors of rectangular plate and cylindrical shell. A discrete-layer shear deformation laminated plate theory was used (He, 1995) to calculate the thermal stresses and deflection of symmetric and anti-symmetric cross-ply plates without shear correction factors. The DQ method (Bert and Malik, 1996) was reviewed in the field of computational mechanics. Some numerical results of displacements and stresses were provided (Shimpi and Ghugal, 2001) for the analysis of trigonometric shear deformation theory in two-layered cross-ply beams. The thermal bending and thermal vibration have been successively analyzed, respectively, by the GDQ method, for the laminated orthotropic plates (Hong and Jane, 2003a,b; Jane and Hong, 2000).

In this present paper, the GDQ method is used to calculate and make an original contribution to study the effect of shear deformation on laminated anisotropic anti-symmetric plates under the thermal and bending loads. Some of the computational GDQ results are compared with others method of research solutions, e.g. closed-form solution, finite element method, a sinusoidal type shape function, and discrete-layer theory solution.

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2. Formulation

We considered a laminated plate with the thickness h^*, the length a and b in the x,y direction, respectively. The laminated plate was considered subjected to a uniformly distributed pressure load q, the temperature difference $\Delta T = T_0(x,y) + (z/h^*)T_1(x,y)$ thermal effect and under in-plane distributed forces p_1, p_2. The static displacements u, v and w of laminate in the x, y and z directions, respectively, were given in the YNS form as follows (Yang et al., 1966; Whitney, 1987):

$$u = u^0(x,y) + z\psi_x(x,y), \quad v = v^0(x,y) + z\psi_y(x,y), \quad w = w(x,y) \tag{1}$$

where u^0 and v^0 are tangential displacements, w is transverse displacement of the middle-plane, ψ_x and ψ_y are the shear rotations.

The static equilibrium differential equations could be represented in terms of displacements u^0, v^0, w, shear rotations ψ_x, ψ_y and the shear correction parameters, for the more details on the differential equations could be referred by the introductions of paper (Hong and Jane, 2003a,b).

We considered a homogeneous, anisotropic, anti-symmetric, rectangular laminated plate subjected to thermal load with sinusoidal temperature distribution ($T_0 = 0, T_1 = \overline{T}_1 \sin(\pi x/a)\sin(\pi y/b)$) and without thermal shear effect ($\alpha_{xy} = 0$). The in-plane stresses $\sigma_x, \sigma_y, \sigma_{xy}$ and the inter-laminar shear stresses σ_{yz}, σ_{xz} including thermal effects were calculated from the transformed reduced stiffness and shear strains $\varepsilon_{yz}, \varepsilon_{xz}$ (Whitney, 1987).

We considered the following two types of laminate with four edges simply supported boundary conditions.

When the cross-ply ($A_{16} = A_{26} = A_{45} = 0, D_{16} = D_{26} = 0, B_{11}, B_{22} \neq 0$) was considered, the following boundary conditions were used.

$$\begin{aligned} &\text{At } x = 0 \quad \text{and} \quad a: \ \partial u^0/\partial x = v^0 = w = \partial\psi_x/\partial x = \psi_y = 0 \\ &\text{At } y = 0 \quad \text{and} \quad b: \ u^0 = \partial v^0/\partial y = w = \psi_x = \partial\psi_y/\partial y = 0 \end{aligned} \tag{2}$$

When the angle-ply ($A_{16} = A_{26} = A_{45} = 0, D_{16} = D_{26} = 0, B_{16}, B_{26} \neq 0$) was considered, the following boundary conditions were used.

$$\begin{aligned} &\text{At } x = 0 \quad \text{and} \quad a: \ B_{16}\partial u^0/\partial y + D_{11}\partial\psi_x/\partial x = v^0 = w = A_{11}\partial u^0/\partial x + B_{16}\partial\psi_x/\partial y = \psi_y = 0 \\ &\text{At } y = 0 \quad \text{and} \quad b: \ u^0 = B_{26}\partial v^0/\partial x + D_{22}\partial\psi_y/\partial y = w = \psi_x = A_{22}\partial v^0/\partial y + B_{26}\partial\psi_y/\partial x = 0 \end{aligned} \tag{3}$$

where $(A_{ij}, B_{ij}, D_{ij}) = \int_{\frac{-h^*}{2}}^{\frac{h^*}{2}} \overline{Q}_{ij}(1,z,z^2)dz, (i,j = 1,2,6)$

$$A_{i^*j^*} = \int_{\frac{-h^*}{2}}^{\frac{h^*}{2}} k_\alpha k_\beta \overline{Q}_{i^*j^*} dz, \quad (i^*, j^* = 4,5; \alpha = 6 - i^*, \beta = 6 - j^*)$$

In which k_α, k_β are the shear correction coefficients, $\overline{Q}_{ij}$ is the transformed reduced stiffness, $\overline{Q}_{i^*j^*}$ is the shear transformed reduced stiffness that denotes as follows.

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{44} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{yz} \\ \varepsilon_{xz} \end{Bmatrix} \tag{4}$$

3. Computational procedures

Two-dimensional GDQ method could be applied to obtain the static discretized equations, and the following non-dimensional parameters were used:

$$X = x/a, \quad Y = y/b, \quad U = u^0/a, \quad V = v^0/b, \quad W = 10h^*w/(\alpha_x \overline{T}_1 a^2)$$

The implement procedures of discretized equation could be referred from the presentations of paper (Shu and Du, 1997; Hong et al., 2005). The static discretized equation could be rewritten in the following matrix form:

$$[A]\{W^*\} = \{B\} \tag{5}$$

where $[A]$ is a dimension of N^{**} by N^{**} coefficient matrix ($N^{**} = (N-2) \times (M-2) \times 5$) containing the GDQ approximated weighting parameters $A_{i,l}^{(1)}, A_{i,l}^{(2)}, B_{j,m}^{(1)}, B_{j,m}^{(2)}$, etc. $\{W^*\}$ is a N^{**}th order unknown column vector and $\{B\}$ is a N^{**}th order row vector, in the $N \times M$ grid points.

4. Some numerical results and discussions

We considered the two typical material types of advanced fiber-reinforced laminate with the parameters $G_{12} = G_{13}, v_{12} = v_{13}$ and $k_\alpha k_\beta = 5/6$. The two typical laminated plates were made of identical unidirectional plies with the anisotropic elastic properties, and given as follows (Reddy and Hsu, 1980):