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A quasi-velocity-based nonlinear controller for rigid manipulators

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ABSTRACT

The problem of position control in the operational space of a robot manipulator is addressed in the paper. The proposed controller is based on equations of motion expressed in terms of normalized generalized velocity components (NGVC) which result from decomposition of the manipulator inertia matrix. The sufficient conditions for global exponential stability of the system under the controller are given. It is shown that using the controller an further insight into the system dynamics is possible. The proposed control algorithm is tested via simulation on a spatial manipulator with three degrees of freedom.

1. Introduction

The aim of the task space position control relies on moving the manipulator from its arbitrary initial configuration to a desired end-effector position and orientation in the operational space. There exist well-known control algorithms based on inverse dynamics control (Canudas de Wit et al., 1996; Sciavicco and Siciliano, 1996) or sliding mode control (Slotine and Li, 1987, 1991) which solve this problem. In fact, they are an extension of control strategies used in the manipulator joint space (Sciavicco and Siciliano, 1996; Slotine and Li, 1987; Spong and Vidyasagar, 1989). From practical point of view to track the position of the end-effector of the manipulator is more convenient than the joint position tracking. The motion control problem in the manipulator operational space is still investigated (Kelly and Moreno, 2005). The control algorithms are based on second-order differential equations of motion which are strongly nonlinear and coupled.

The modern approach considers dynamics of mechanical systems using quasi-velocities and differential geometry (Kwatny and Blankenship, 2000). The obtained first-order equations of motion are the Poincaré's form of Lagrange's equations. There exist various quasi-velocities arising from decomposition of the system inertia matrix (Hurtado, 2004; Jain and Rodriguez, 1995; Junkins and Schaub, 1997; Loduha and Ravani, 1995; Sovinsky et al., 2005). The quasi-velocities, in contrast to the generalized velocities, contain the kinematic as well as the dynamic parameters of the system. There are also some control algorithms based on the quasi-velocity approach e.g. (Herman, 2005; Kwatny and Blankenship, 2000; Schaub and Junkins, 1997). The normalized generalized velocity components (NGVC) considered here were described in Herman (2005, 2006) and they are a kind of quasi-velocities. They are an useful form of the generalized velocity components introduced by Loduha and Ravani (1995).

The objective of this paper is to present a control algorithm for manipulators in the operational space using a modified non-adaptive sliding mode control law proposed originally by Slotine and Li (1987, 1991). The introduced and analyzed here trajectory tracking controller is based on the NGVC vector. The presence of the manipulator dynamics in the controller

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Nomenclature

$\mathcal{N}$ number of the degrees-of-freedom
$\theta, \dot{\theta}, \ddot{\theta} \in R^{\mathcal{N}}$ vectors of generalized positions, velocities, and accelerations, respectively
$M(\theta) \in R^{\mathcal{N} \times \mathcal{N}}$ system inertia matrix
$C(\theta, \dot{\theta})\dot{\theta} \in R^{\mathcal{N}}$ vector of Coriolis and centrifugal forces in classical equations of motion arising from the relationship $C^*(\theta, \dot{\theta}) = C(\theta, \dot{\theta})\dot{\theta} = (\dot{M}\dot{\theta} - \frac{1}{2}\dot{\theta}^{\mathrm{T}} M_{\theta} \dot{\theta})$, where the expression $\dot{\theta}^{\mathrm{T}} M_{\theta} \dot{\theta}$ is the column vector $\mathrm{col}(\dot{\theta}^{\mathrm{T}} M_{\theta_k} \dot{\theta})$ ($M_{\theta_k} = \frac{\partial M}{\partial \theta_k}$ means the partial derivative of the inertia matrix $M(\theta)$ with respect to the joint coordinate θ_k) and $\dot{M}$ is the time derivative of the matrix $M(\theta)$ (derivation of the term $C^*(\theta, \dot{\theta})$ can be found e.g. in Koditschek (1985))
$G(\theta) \in R^{\mathcal{N}}$ vector of gravitational forces in classical equations of motion
$f(\dot{\theta}) = F\dot{\theta} \in R^{\mathcal{N}}$ vector of forces due to viscous friction which depends on the joint velocity vector $\dot{\theta}$ where $F = \mathrm{diag}\{F_1, \ldots, F_{\mathcal{N}}\}$ is a positive definite diagonal matrix containing the damping coefficients for all joints
$Q \in R^{\mathcal{N}}$ vector of generalized forces
$N \in R^{\mathcal{N} \times \mathcal{N}}$ diagonal system inertia matrix arising from decomposition of the inertia matrix $M(\theta)$ according to the method presented in Loduha and Ravani (1995)
$\vartheta, \dot{\vartheta} \in R^{\mathcal{N}}$ vector of quasi-velocities, i.e. normalized generalized velocity components (NGVC) and its time derivative, respectively
$\Upsilon = \Upsilon(\theta) \in R^{\mathcal{N} \times \mathcal{N}}$ upper triangular velocity transformation matrix arising from decomposition of the inertia matrix $M(\theta)$ using the method described in Loduha and Ravani (1995)
$\Phi = \Phi(\theta) \in R^{\mathcal{N} \times \mathcal{N}}$ upper triangular velocity transformation matrix in terms of NGVC
$C_{\vartheta}(\theta, \vartheta)\vartheta \in R^{\mathcal{N}}$ vector of Coriolis and centrifugal forces in terms of NGVC
$G_{\vartheta}(\theta) \in R^{\mathcal{N}}$ vector of gravitational forces in terms of NGVC
$f_{\vartheta}(\theta, \dot{\theta}) \in R^{\mathcal{N}}$ vector of friction damping forces in terms of NGVC
$\varpi \in R^{\mathcal{N}}$ vector of quasi-forces in terms of NGVC
$(.)^{\mathrm{T}}$ transpose operation

velocity gain matrix leads to faster position and orientation error convergence, and the kinetic energy reduction than the well-known algorithm reported in the literature. It will be shown that the state space origin of the system under the proposed controller is globally exponentially stable in the sense of Lyapunov. In contrast to reference (Herman, 2005), the controller considered here works in the manipulator operational space instead of in its joint space. Moreover, the control algorithm uses the full system dynamics because the new inertia matrix is the identity one. The second goal is to show that information obtained from the proposed NGVC controller is useful for evaluation of the system dynamics. Thus, the proposed approach can be considered as an auxiliary tool in the manipulator design process.

The paper is organized as follows. In the third section first-order equations of motion in terms of NGVC are presented. The proposed task-oriented controller is shown in the fourth section. Simulation results for a spatial 3 df manipulator are presented in the fifth section. The last section presents conclusions.

2. Manipulator dynamics in terms of NGVC

The equations of motion of a $\mathcal{N}$ degree-of-freedom manipulator can be written as Sciavicco and Siciliano (1996):

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) + f(\dot{\theta}) = Q. \tag{1}$$

The above equations are derived using the Lagrange's formulation. There exist various methods which allow us to decompose the matrix $M(\theta)$ in order to obtain a diagonal inertia matrix. The equations of motion are known as the Poincaré's equations. In this paper the inertia matrix decomposition method described by Loduha and Ravani (1995) was chosen because of its easy implementation for manipulator control algorithms.

According to the assumed method the matrix $M(\theta)$ is decomposed into three matrices (the rate transformation matrix Υ is invertible for robot manipulators because the inertia matrix is positive definite and symmetric), i.e.:

$$M(\theta) = (\Upsilon^{\mathrm{T}})^{-1} N \Upsilon^{-1}. \tag{2}$$

Denoting now the transformation matrix $\Phi = N^{\frac{1}{2}} \Upsilon^{-1}$ (Herman, 2005, 2006) one can give the inertia matrix $M(\theta)$ in the following form (where $\Phi = \Phi(\theta)$):

$$M(\theta) = \Phi^{\mathrm{T}} \Phi. \tag{3}$$

Substituting (3) into (1) one obtains the equation containing the matrix Φ. Multiplying next both sides of the new equation by $(\Phi^{\mathrm{T}})^{-1}$ (the matrix Φ is invertible for robot manipulators) one gets:

$$\Phi\ddot{\theta} + \dot{\Phi}\dot{\theta} + (\Phi^{\mathrm{T}})^{-1} C(\theta, \dot{\theta})\dot{\theta} - \dot{\Phi}\dot{\theta} + (\Phi^{\mathrm{T}})^{-1} G(\theta) + (\Phi^{\mathrm{T}})^{-1} f(\dot{\theta}) = (\Phi^{\mathrm{T}})^{-1} Q. \tag{4}$$