Mechanism and Machine Theory 88 (2015) 16–30

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Kinematic analysis of geared robotic mechanism using Matroid and T–T graph methods

S. Vahid Amirinezhad, Mustafa K. Uyguroğlu *

Department of Electrical and Electronic Engineering, Eastern Mediterranean University, Gazimağusa, North Cyprus

A R T I C L E I N F O

Article history:
Received 22 September 2014
Received in revised form 23 January 2015
Accepted 15 February 2015
Available online 2 March 2015

Keywords:
Matroid method
T–T graph method
Geared robotic mechanisms (GRMs)
Kinematic analysis

A B S T R A C T

In this paper, the kinematic structure of the geared robotic mechanism (GRM) is investigated with the aid of two different methods which are based on directed graphs and then these methods are compared, accordingly. One of the methods is Matroid method developed by Talpasanu and the other is Tsai–Tokad (T–T) graph method developed by Uyguroğlu and Demirel. The findings show that the kinematic structure of the geared robotic mechanism can be represented by directed graphs and angular velocity equations of the mechanisms can be systematically obtained from the graphs. The advantages and disadvantages of both methods are demonstrated in relation to each other.

1. Introduction

In recent years, a number of methods concerning the general approach to kinematic analysis [1–10], the structure of geared mechanisms [11–14] and the kinematic and dynamic analyses of geared systems [15–33] have been developed.

Some of the methods developed for kinematic and dynamic analyses of mechanical systems are based on graph theory. Non-oriented and oriented graphs were used for this purpose. The non-oriented graph technique is mainly used for the kinematic analysis of robotic bevel-gear trains [22–25]. The oriented graph technique has been used for electrical circuits and other types of lumped physical systems including mechanical systems in one-dimensional motion since the early sixties [2,5,7,8]. Chou et al. [9] used these techniques for three-dimensional systems. Recently, Tokad developed a systematic approach, the so-called Network Model Approach, for the formulation of three dimensional mechanical systems [10], and Uyguroğlu and Tokad extended this approach for the kinematic and dynamic analyses of spatial robotic bevel-gear trains [26,27]. More recently, the oriented and non-oriented graph techniques were compared by Uyguroğlu and Demirel [28], and the advantages of the oriented graph over the non-oriented graph were demonstrated using the kinematic analysis of bevel-gear trains. In order to overcome the weaknesses of the methods developed by Tsai and Tokad, both methods were combined and the so called T–T Graph method was introduced [29]. Most recently, Talpasanu [30] and Talpasanu et al. [31–33] developed the Matroid method for the kinematic analysis of geared mechanisms based on directed graph as well.

In this paper, the kinematic structures of the GRM are investigated with the aid of the Matroid method and the T–T graph method. Section 2 describes the kinematic structure of the GRM. The kinematic analysis of the GRM using the Matroid and T–T graph methods are given in Sections 3 and 4, respectively. Since both methods employed directed graph, the similarities and the differences are shown and the advantages of each system are specified.

* Corresponding author.
E-mail addresses: vahid.amirinezhad@cc.emu.edu.tr (S. Vahid Amirinezhad), mustafa.uyguroglu@emu.edu.tr (M.K. Uyguroğlu).

http://dx.doi.org/10.1016/j.mechmachtheory.2015.02.002
0094-114X/

Nomenclature

$\mathbf{\Gamma}^0$	incidence node-edge matrix
$\mathbf{\Gamma}$	reduced incidence node-edge matrix
$\mathbf{l}^0_{c,k}$	distance vector between c and k
α	joint twist angle
θ	joint angle
$\dot{\theta}, \omega$	angular velocity
$\dot{\theta}_{i/j}, \omega_{ij}$	angular velocity of link i with respect to link j
$\boldsymbol{\omega}^0_n$	absolute angular velocity vector
$\mathbf{r}^0_{c,k}$	position vector of z_k (second vector in screw)
${}^{0}_{c,k}$	screw matrix
$\mathbf{u}$	unit vector along the local frame z axis
$\mathbf{u}^0_k$	representation of $\mathbf{u}$ with respect to base coordinate frame
$\hat{\mathbf{s}}^0_{c,k}$	twist about a screw, a 6×1 column matrix attach to each turning or gear pair
$\mathbf{P}_{c,t}$	position matrix
$\mathbf{D}_{0,k}$, ${}^0\mathbf{R}_k$	rotational transformation matrix
c	number of gear pairs
$\mathbf{C}$	cycle basis matrix
C_i	fundamental cycle
d_i	pitch diameter of gear i
E	degree of freedom
$\mathbf{G}$	matrix associated with tree branches (arcs)
$\mathbf{G}^*$	matrix associated with tree chords
k	number of joints
n	number of links
n_{head}	end node of directed edge
N_i	teeth number of gear i
n_{ij}	teeth ratio of gears i and j, N_i/N_j
n_{tail}	starting node of directed edge
r	rank of cycle basis matrix
t	number of turning pairs
$\mathbf{U}$	identity matrix
$\mathbf{Z}$	path matrix

2. Geared robotic mechanism

GRMs are closed-loop configurations which are used to reduce the mass and inertia of the actuators' loads. Gear trains in GRMs are employed so that actuators can be placed as closely as possible to the base. Fig. 1 shows functional schematic of the GRM. It has 3 degrees of freedom and the end-effector may have spatial motion (3 dimensional) since this mechanism can generate two rotations about two intersected axes and one rotation of end-effector about its axis. In this mechanism, 4, 5, and 6 are sun gears (input links), 1 and 2 are carriers and 3, 7′, and 7″ are planet gears. It is observed that links and joints (gear train) are used to transmit the rotation of the inputs to the end-effector. The motion of the end-effector is produced by links 4, 5, and 6 as inputs. The end-effector is attached to link 3 and carried by link 2. *M1*, *M2 and M3* are actuators. The rotation of link 3 is caused by *M3* through 6 and 7 links and the rotation of links 1 and 2 is determined by *M1* through link 4 and *M2* through link 5, respectively.

3. Matroid method

In this section, the Matroid method [30] is applied to the sample geared robotic mechanism (GRM) to obtain kinematic equations. First, its digraph is sketched and corresponding matrices are obtained. Then, relative angular velocities are calculated by using these matrices and screw theory.

3.1. Associated digraph and corresponding matrices

The mechanism in Fig. 1 consists of $n = 7$ links, $t = 7$ turning pairs and $c = 4$ gear pairs. Note that $k = 11$ is total number of joints (i.e. $k = t + c$ and $t = n$). The following labeling, which is assigned to links and joints of sample mechanism, is used in the Matroid method [31]:

- 0 is assigned to ground link.
- 1, 2, 3, 4, 5, 6, and 7 are assigned to gears and carriers.