Mechanism and Machine Theory 85 (2015) 64–81

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Static and dynamic stiffness analyses of cable-driven parallel robots with non-negligible cable mass and elasticity

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A R T I C L E I N F O

Article history:
Received 18 March 2014
Received in revised form 19 September 2014
Accepted 28 October 2014
Available online 25 November 2014

Keywords:
Cable-driven parallel robot
Sagging cable model
Static stiffness
Dynamic stiffness
Vibration

A B S T R A C T

This paper focuses on the stiffness analysis of cable-driven parallel robots (CDPRs), including the static stiffness and the dynamic stiffness analyses. Static and dynamic cable models are introduced considering the effect of both cable mass and elasticity. Based on these models, the static stiffness of CDPRs is evaluated by the variation of the end-effector pose error, and the dynamic stiffness of CDPRs is analyzed by identifying the robot natural frequencies. Simulations and experiments are made on a 6-DOF prototype to validate the theoretical models. Comparison with other methods available in literature is presented. Results show the important effect of cable mass and elasticity on the static and dynamic stiffness of CDPRs.

1. Introduction

Cable-driven parallel robots (CDPRs) are a special variant of traditional rigid-link parallel robots. There are some advantages to use flexible cables instead of rigid links, such as high dynamics due to small moving mass, large workspace, and low cost [1]. However, as cables present the particularity of not being rigid and are only able to act in tension, the stiffness of CDPRs becomes a vital concern [2,3]. Stiffness performances have a significant effect on the static and dynamic behaviors of CDPRs, such as kinematics, positioning accuracy, force distribution, vibration and control [1,4]. Deficient static stiffness can decrease the positioning accuracy of CDPRs, and bad dynamic stiffness characteristics can lead to vibration and long settling time. Although stiffness has been well studied in the last few decades for rigid-link parallel robots [1,4–8], there is little literature on the stiffness problem of CDPRs.

When it comes to the static stiffness analysis of CDPRs, an important issue is cable modeling. Many studies used linear or non-linear spring as cable model [9–17]. This approach only considers the elasticity along cable axis and assumes cable as massless spring. This assumption is not accurate, especially for CDPRs with heavy and/or long-span cables. In fact, the axial cable stiffness is not the only source of the static stiffness. Sag-introduced stiffness should also be considered. Another well known model is the static sagging cable model derived from civil engineering [18]. It is used in several previous researches [2,3,19–21]. The sagging cable model considers the effect of cable mass and elasticity. It is more accurate than the spring cable model in the static stiffness analysis of CDPRs. In previous researches, the effect of cable sag on the static stiffness of CDPRs is verified [3,19,21]. However, to our best knowledge, the verification is limited to numerical investigations. Experimental verification of the static stiffness is only performed on a single sagging cable [18,19] but not on CDPRs.

Another issue for the static analysis is the index of stiffness performance evaluation. Most studies [9,10,21] use Cartesian stiffness matrix or its mathematical properties (such as determinant, trace, norm, and etc) as evaluation indices. For massless cable

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http://dx.doi.org/10.1016/j.mechmachtheory.2014.10.010

assumption, the static stiffness of CDPRs only depends on the axial stiffness of cables. In linear-elastic range, the axial stiffness is independent of cable forces. Thus the Cartesian stiffness matrix is independent of the external wrench applied to the end-effector. It is easy to compute the Cartesian stiffness matrix through the Jacobian matrix of CDPRs. However, with non-negligible cable mass, the cable profile between two attachment points is not a straight line but a sagging curve. So the direction of cable force is not along the chord of the curve but along the tangent line of the curve. In this case, Jacobian matrix cannot be used to calculate the Cartesian stiffness matrix, and partial differential equations should be employed instead. This increases calculation complexity. Furthermore, taking the cable sag into consideration, the stiffness of cables is relevant to cable forces and thus depends on the external wrench. Previous researches do not present the variation of the static stiffness with the external wrench.

Some applications of CDPRs require high performances, especially the dynamic performances. For examples: the ultrahigh speed FALCON robot [11,12], the wind-induced vibration problem of the large radio telescope [22] and the wind tunnels [23]. Vibration can be induced by initial position and velocity of the end-effector, wind disturbance, and/or friction of the cables around fixed pulleys [24]. Vibration can affect the positioning accuracy of the end-effector, and bring fluctuation on the trajectory. These applications lead to researches on the dynamic stiffness of CDPRs in recent years. Natural frequency is widely used in literature as an index for the dynamic stiffness evaluation of CDPRs [19,24–28]. Most of these researches [25,26,28] only consider cable elasticity, while neglect cable mass. Although cable mass is considered for the static analysis in [19], it is ignored in the computation of robot natural frequencies. As a matter of fact, in many situations, both cable mass and elasticity will affect system dynamics by changing the value of natural frequencies and/or adding new resonances. Therefore, both cable mass and elasticity should be taken into consideration for the dynamic analysis of CDPRs. The finite element cable model in [24,27] considers the effect of cable mass. This method uses distributed mass points and ideal lines between them to simulate continuous cable, but it leads to a system with partial differential equations. In addition, the accuracy of finite element method depends on the number of elements. To ensure a good accuracy will result in further computational complexity.

This paper focuses on the static and dynamic stiffness analyses of CDPRs. Static sagging cable model is introduced. This cable model considers cable mass and elasticity, and describes the static cable profile with a set of non-linear equations. Cable stiffness contains both axial flexibility and sag-introduced flexibility. Based on this sagging cable model, the variation of pose error with external load is used as an index for evaluating the static stiffness performance of CDPRs. The pose error of the end-effector can be calculated through the kinematic model of CDPRs. Based on this method, the static stiffness of CDPRs is analyzed by both simulations and experiments on a 6-DOF CDPR prototype. The effect of cable sag on the static stiffness is validated by experiment for the first time. The variation of the static stiffness with external load is presented.

A new dynamic stiffness model of CDPRs is proposed in this paper. This model is based on the Dynamic Stiffness Matrix (DSM) method. DSM is used to solve the vibration problems of structures. It is often regarded as an exact method, because DSM is based on the exact shape functions obtained from the exact solution of the element differential equations [29]. This method provides better accuracy compared with finite element method. Firstly, the dynamic stiffness matrix of a single cable proposed by [30,31] is introduced in this paper. This dynamic cable model considers the effect of cable mass and elasticity. Then the dynamic stiffness matrix of CDPRs is deduced, considering the coupling between end-effector vibration and cable vibration. The dynamic response functions of CDPRs are achieved to identify the system natural frequencies. In addition, experiments on a 6-DOF CDPR prototype are performed to verify the proposed dynamic stiffness model. The Frequency Response Functions of the prototype are calculated, and natural frequencies are identified. Experimental results are also compared with other methods available in literature.

This paper is organized as follow. Static cable model and dynamic cable model are firstly introduced in Section 2. Then the static and dynamic stiffness analyses of CDPRs are presented in Section 3. In this section, the kinematic model of CDPRs is set up. Based on this model, the pose error of CDPRs is defined, and the variation of the end-effector pose error with the external load is regarded as an index for the static stiffness evaluation. The dynamic stiffness matrix of CDPRs is formulated, and dynamic response functions are deduced to identify the natural frequencies. Section 4 presents a 6-DOF CDPR prototype. Simulations and experiments are carried out to investigate the static and dynamic stiffness performance of the prototype. Section 5 analyzes the stiffness characteristics of the prototype over its workspace. Applications of the proposed method on the design procedures of suspended CDPRs and non-suspended CDPRs are discussed. Finally, conclusions are made in Section 6.

2. Cable modeling

Cable modeling is the basis of the stiffness analysis of CDPRs. In this section, the static sagging cable model and the DSM cable model are introduced. Compared with massless spring model, the proposed cable models considering both cable mass and elasticity are more accurate in describing the static and dynamic cable behavior.

2.1. Static cable model

The static sagging cable model, also known as elastic catenary model, considers the effect of both cable mass and elasticity. It gives the static cable profile by a set of non-linear equations. This model has been studied and used in civil engineering since 1930s [18]. However, it is quite new in the analysis of CDPRs [3]. In addition, this cable model is the theoretical basis of this paper. It is necessary to briefly introduce this model with variables familiar to robotics.