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A general method to determine compatible orientation workspaces for different types of 6-DOF parallel manipulators

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ABSTRACT

This paper presents a general method to develop a compatible orientation workspace for a 6-DOF parallel manipulator. The workspace boundary for any type of manipulator can be determined using the proposed method, if the equations for inverse kinematics can be developed. The workspace boundary can be developed by solving the equations, but the results show that a search technique that uses the bisection method is more efficient, if the equations are 4th or higher degree polynomials. In general, a workspace can be developed in less than 5 min, using a personal computer. The effect of the size of the platform, the passive joint limits, the link interactions and singularity on the shape and size of a workspace are also determined.

1. Introduction

An orientation workspace is commonly used to evaluate the degree of possible rotations of a platform about the tool center point (TCP) on the platform. It is the set of all attainable orientations of the platform about a fixed point. The workspace is a three-dimensional subspace, with a boundary that consists of several two-dimensional patches, each of which is generated by one constraint equation from the actuated joint limits, the passive joint limits or the link interactions. This paper directly determines the exact equations that are required to develop a compatible orientation workspace, which is defined as the set of orientations that are reached through a continuous motion, starting from an initial configuration.

The orientation workspace (herewith termed the workspace) for parallel manipulators has been intensively studied over the past three decades and a workspace can be determined using many existing methods [1–19]. Most studies use discretization methods to develop the workspace of a Stewart–Gough manipulator. If inverse kinematics yields admissible solutions (the real solutions with all of the joint displacements within their joint limits) for one orientation, then the orientation is in the theoretical workspace. However, this orientation may not be in the compatible workspace. Fig. 1 shows two configurations for a Stewart–Gough manipulator. Two compatible workspaces can be developed using the two configurations as the starting points. A point in one workspace cannot reach another point in the other workspace via a continuous motion. A real compatible workspace cannot be developed without considering the effect of spherical joint limits and link interactions. In general, the distance between two limbs is used to check whether the two limbs interfere. In theory, a compatible workspace can be determined using discretization methods. However, if link interactions do not occur at two neighboring points, this does not guarantee that the two points are in the same compatible workspace, if a larger step is chosen for discretization. Link interactions can occur somewhere between the two points, so the two points belong to

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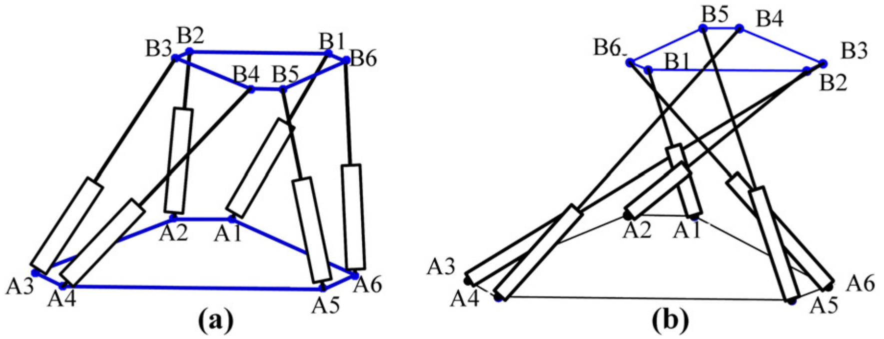

Fig. 1. Two inadmissible configurations.

two different compatible workspaces. A very small step must be used for discretization, to develop a compatible workspace, which means that inverse kinematics must be solved and it is necessary to check whether the solutions satisfy all of the possible constraints at a lot of sample points, to develop a compatible workspace. Therefore, the development of a compatible workspace using a discretization method is an extremely time-consuming task. Developing a workspace boundary via a continuous motion is a more efficient and reliable approach. Jiang and Gosselin presented an analytical method to evaluate the workspace for a Stewart–Gough manipulator. The method first develops all possible boundary curves on a cross-section and then selects the real boundary curves, to obtain the workspace [15]. Although the proposed method does not directly determine the boundary of a workspace, a theoretical workspace (a workspace developed without considering the effect of passive joint limits and link interactions) is obtained in about 29 s, because the possible boundary curves that are generated by actuator joint limits are developed by solving quadratic equations. However, the obtained workspace is not a real workspace, if any spherical joint reaches its joint limits and it is not a compatible workspace if link interactions occur.

In general, the workspaces that are developed using existing methods are simple workspaces that have only one region on a cross-section of the workspace. For a manipulator with a relatively small platform, spherical joint limits and link interactions can generate some boundary patches, so it can have a complicated workspace, with two or more regions on a cross-section. One new region can emerge (or one old region can disappear), or one region can separate into two regions (or two regions can merge into one region) on the next cross-section, during the development process. Therefore, the development of a compatible workspace, especially a complicated workspace, using an analytical method, is a difficult and time-consuming task. For Stewart–Gough manipulators, the methods used in a previous study can directly determine workspace boundaries by solving constraint equations that are developed from actuator joint limits, spherical joint limits and link interactions. In the development process, inverse kinematics must be solved and it is necessary to check whether any kinematic constraint is violated. The possible changes in the boundaries between two neighboring cross-sections are predicted by solving different sets of nonlinear equations. This approach is quite time-consuming, and it is not applicable to other types of 6-DOF parallel manipulators [20].

This work presents a general method to develop compatible workspaces for different types of 6-DOF parallel manipulators. The constraint equations for these manipulators can be nonlinear equations from spherical joint limits or link interactions, quadratic equations, biquadratic equations, 8th degree polynomials, or higher degree polynomials. The boundaries that are derived from quadratic equations are determined by solving the equations. For other higher degree polynomials and nonlinear equations, the bisection method is employed to develop the related boundary curves on a cross-section of the workspace [21]. Rules for the detection of the possible

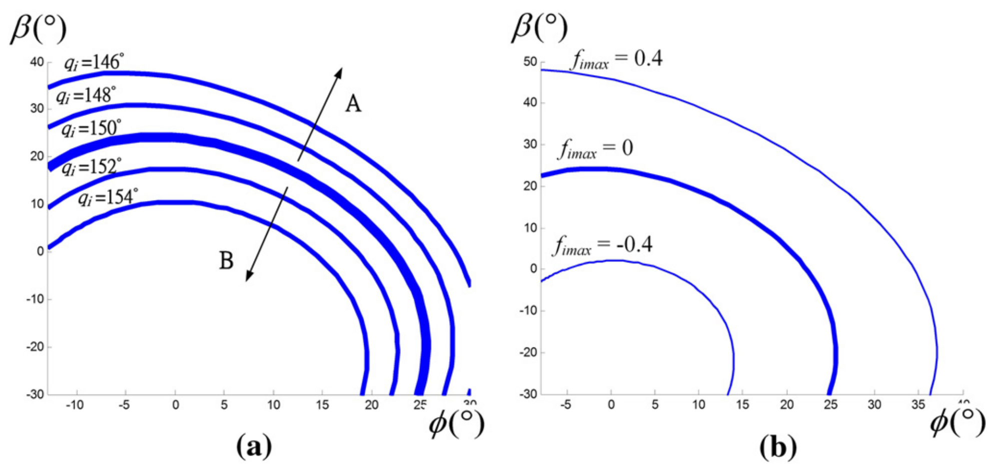

Fig. 2. Admissible region and inadmissible region.