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From sliding–rolling loci to instantaneous kinematics: An adjoint approach

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ABSTRACT

The adjoint approach has proven effective in studying the properties and distribution of coupler curves of crank-rocker linkages and the geometry of a rigid object in spatial motion. This paper extends the adjoint approach to a general surface and investigates kinematics of relative motion of two rigid objects that maintain sliding–rolling contact. We established the adjoint curve to a surface and obtained the fixed-point condition, which yielded the geometric kinematics of an arbitrary point on the moving surface. After time was taken into consideration, the velocity of the arbitrary point was obtained by two different ways. The arbitrariness of the point results in a set of overconstrained equations that give the translational and angular velocities of the moving surface. This novel kinematic formulation is expressed in terms of vectors and the geometry of the contact loci. This classical approach reveals the intrinsic kinematic properties of the moving object. We then revisited the classical example of a unit disc rolling–sliding on a plane. A second example of two general surfaces maintaining rolling–sliding contact was further added to illustrate the proposed approach.

1. Introduction

In classical differential geometry, the adjoint approach is used to study the properties of a curve or a surface via its companion curve or surface [1,2]. For example, the properties of an involute and evolute of a curve are studied using the geometry of the initial curve. Another example is the Bertrand curves that have common principal normal lines [3]. The famous cycloid is the locus of points traced out by a point on a circle that rolls without sliding along a straight line, where the circle is said to be adjoint to the straight line.

The adjoint approach has been applied to mechanical engineering, for example gear mesh [4]. Wang and Xiao [5] extended the adjoint approach to investigating the coupler-curve distribution of crank-rocker linkages. The study of the moving centrode adjoint to the fixed centrode concisely revealed the distribution law of various shapes of coupler curves. They also applied the approach to the moving axodes adjoint to the fixed axodes, revealing the intrinsic properties of a point trajectory, a line trajectory, and characteristic lines on the moving body [6–8].

The sliding–spinning–rolling motion occurs naturally in many systems such as a robotic hand manipulating an object [9–11], the interaction between wheeled vehicles and the ground [12,13], gear and cam transmission [14–16], and biomechanics [17,18]. Developing the kinematic relation between the relative objects facilitates the subsequent dynamics or control of the systems.

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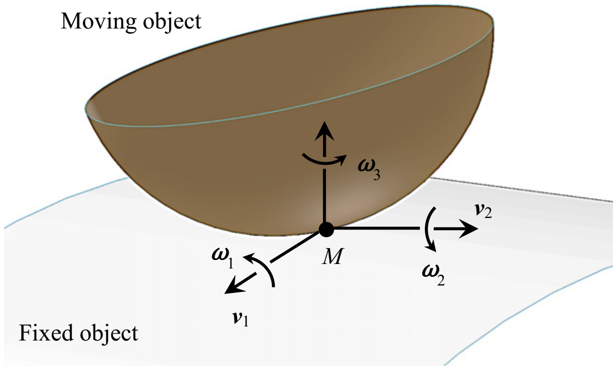

Fig. 1. The two translational sliding DOFs, $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, at the contact point and three rotational DOFs, $\boldsymbol{\omega}_1$, $\boldsymbol{\omega}_2$, and $\boldsymbol{\omega}_3$, about the contact point.

The relative motion between two rigid objects that maintain sliding–rolling contact is a five degrees-of-freedom (DOFs) sliding–spinning–rolling motion, which can be decomposed into two translational sliding DOFs, $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, at the contact point and three rotational DOFs, $\boldsymbol{\omega}_1$, $\boldsymbol{\omega}_2$, and $\boldsymbol{\omega}_3$, about the contact point, as in Fig. 1.

Previous literature on sliding–spinning–rolling motion either restricted the shapes of objects to flat, sphere, or restricted the types of relative motion to rolling contact [19–21]. Sliding motion was sometimes singled out for dexterous manipulation [22]. For general sliding–spinning–rolling motion, the two contact points have different rates and directions, making the derivation process complicated and unintuitive. Two formulations [23,24] have far-reaching effects on later development. The former defined one moving point trajectory and two contact trajectories to derive first- and second-order kinematics of sliding–spinning–rolling motion via Taylor series expansion. The latter derived a set of first-order kinematic equations through the velocity relation between three coordinate frames.

The results were applied to manipulations, control, and motion planning. Li, Hsu and Sastry [25] developed a computed torque-like control algorithm for the coordinated manipulation of a multifingered robot hand based on the assumption of point contact models. Sarkar, Kumar and Yun [26] extended Montana's work to include acceleration terms. By using intrinsic geometric properties for the contact surfaces, they showed the explicit dependence on the Christoffel symbols and their time derivatives. Chen [27–31] coined the term “conjugate form of motion” for kinematics of point contact motion between two surfaces and developed a geometric form of motion representation. Han and Trinkle [32] showed all systems variables needed to be included in the differential kinematic equation used for manipulation planning and further studied the relevant theories of contact kinematics, nonholonomic motion planning. Marigo and Bicchi [33] derived analogous equations with Montana's contact equations, but with a different approach that allowed an analysis of admissibility of rolling contact.

It is natural to apply the adjoint approach to study the kinematics of the moving object, since one contact trajectory curve exists on each of the two objects. While the curve on the moving object is produced solely by rolling motion, the one on the fixed object is generated by both sliding and rolling motions. In addition, sliding motion and rolling motion are independent. Hence, there is in general an angle between these two curves.

This paper extends the adjoint approach to a curve adjoint to a general surface by adopting a purely geometric approach based on the moving-frame method [34–36]. The velocity of an arbitrary point is derived in two different ways, which yield a set of eight equations with five variables. Solving this system of overconstrained equations gives the two linear velocities and the three angular velocities.

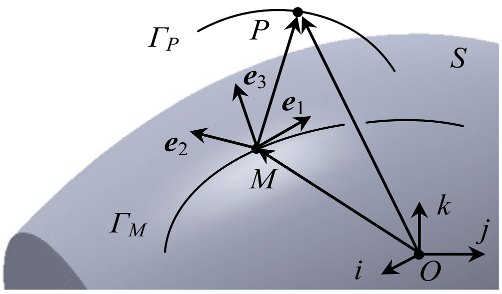

Fig. 2. The curve Γ_P adjoint to the curve Γ_M on the surface S.